THERMAL STABILITY ANALYSIS OF TAPERED FGM ANNULAR PLATES UNDER NONUNIFORM TEMPERATURE DISTRIBUTION - A FINITE ELEMENT APPROACH

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Introduction

Thermal buckling is an important failure mode in plates and shells. Design of such structural elements with minimum weight is of special interest. By an accurate design of thickness distribution, one can get the same required buckling capacity of a plate with considerable weight reduction compared to its uniform thickness counterpart. The subject of thermal stability of homogeneous or FGM circular / annular plates of uniform or variable thickness have been studied by several researchers in recent year (see Refs. [1],[2],[3],[4]).
In this paper, a finite element formulation is developed for analyzing the axisymmetric thermal buckling of FGM annular plates of variable thickness. The plate assumed to obey Kierchhoff plate theory, and thermal load assumed to have a non-uniformly distribution through the radius. Validation of the presented FEM is investigated, and parametric studies are carried out.

Theoretical Formulation

The geometry of a FGM annular plate of variable thickness and the coordinate axes are illustrated in Fig.1. In the FGM plate a material property, P, such as elastic modulus E and coefficient of thermal expansion α, at each side of its midplane is assumed to vary according to a simple power law along the thickness coordinate z, and has symmetry with respect to the midplane, i.e.:

$$P^{(1)}(z) = P_m + (P_c - P_m)(-2z/h(r))^k, \quad -h(r)/2 \le z \le 0,$$
$$P^{(2)}(z) = P_m + (P_c - P_m)(2z/h(r))^k, \quad 0 \le z \le h(r)/2 \qquad (1)$$

where P_m, P_c are properties of metal and ceramic resp., $h(r)$ is the plate thickness as an arbitrary function of the radial position r, and k is the volume fraction index. In order to determine the prebuckling membrane force distribution due to temperature rise in an axisymmetric annular plate, we develop the prebuckling plane elasticity problem. The weak form of the governing differential equation, useful to derive a FEM model, is:

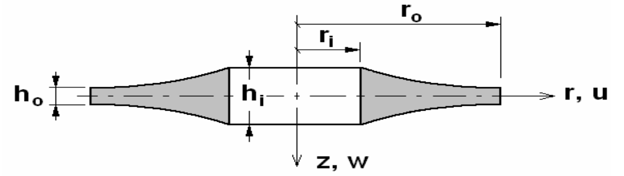


Fig. 1. Geometry of a tapered FGM annular plate.

$$\int_{r_i}^{r_o} \left[a(r) du_0/dr \times d\delta u_0/dr + C(r) u_0 \delta u_0 - \delta u_0 q(r) \right] dr \qquad (2)$$
$$- \delta u_0(r_i) Q_A - \delta u_0(r_o) Q_B = 0$$

in that:

$$a(r) = rA_{11}, \quad C(r) = A_{11}/r - dA_{12}/dr, \quad q(r) = -rdN_r^T/dr,$$
$$-Q_A = a(r)\, du_0/dr\,|_{r_i}, \quad Q_B = a(r) du_0/dr\,|_{r_o} \qquad (3)$$

where $N_r^T = F\, h(r)\Delta T(r)/(1-\nu)$, in that F is a constant expression of material properties of metal and ceramic.
We can also derive the weak form of the D.E. governing the plate thermal buckling using PMPE, as follows:

$$\delta\pi = \int_{r_i}^{r_o} \left[d^2w/dr^2 \quad r^{-1}dw/dr \right] \begin{bmatrix} D & \nu D \\ \nu D & D \end{bmatrix} \begin{Bmatrix} d^2\delta w/dr^2 \\ r^{-1}d\delta w/dr \end{Bmatrix} r\, dr$$
$$-2\int_{r_i}^{r_o} N_r (dw/dr)(d\delta w/dr) r dr = 0 \qquad (4)$$

where $D = \dfrac{(3E_c + E_m k)}{12(1-\nu^2)(k+3)} h^3(r)$ is the plate flexural rigidity.

Finite Element Formulation

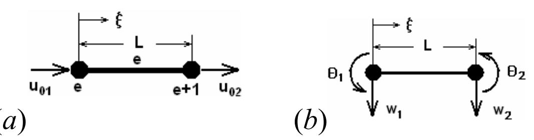
In order to derive the finite element formulation for the plate prebuckling behavior, we make use of the weak form Eq.(2) stated for a typical element in membrane deformation as shown in Fig. 2*a*.. To approximate the element radial displacement function u^e, the linear Lagrange interpolation functions are implemented, i.e.:

$$u_0^e = N_1\, u_{01}^e + N_2\, u_{02}^e \qquad (5)$$

Following the Rayleigh-Ritz procedure, the plane elasticity finite element formulation can be obtained as:

$$[K^e]\{u_0^e\} = \{f^e\} + \{Q^e\} \qquad (6)$$

where the entries of the stiffness matrix $[K^e]$, force vectors $\{f^e\}$ and $\{Q^e\}$ are determined by:



(*a*) (*b*)

Fig. 2. An element with its nodal degrees of freedom in *a*) membrane deformation *b*) transverse deflection.

$$K_{ij}^{e}=\int_{0}^{L}\left\{a(\xi)\frac{dN_i^e}{d\xi}\left(\sum_{j=1}^{2}\frac{dN_j^e}{d\xi}u_{0j}^{e}\right)+C(\xi)N_i^e\left(\sum_{j=1}^{2}N_j^e u_{0j}^{e}\right)\right\}dr$$

$$f_i=\int_{0}^{L}\left\{N_i^e\,q(\xi)\right\}dr\,,\quad Q_i^e=\sum_{j=1}^{2}N_j^e(\xi_i^e)Q_j^e \tag{7}$$

Solving the expanded form of Eq.(6), the displacement vector $\{u_0\}$ obtained as function of the temperature index T_0. Note that $T(r)$ is expressible as $T_0 f(r)$. Then, using the plate kinematic and constitutive equations, the membrane forces will be determined as $N_r=T_0\overline{N}(r)$.

To derive a finite element model for the plate stability equation, we apply Eq.(4) stated for a typical element as shown in Fig. 2*b*., and make use of Hermite family of interpolation functions, to approximate the element displacement function w^e, as:

$$w^e(\xi)=\left[N_1^e\ \ N_2^e\ \ N_3^e\ \ N_4^e\right]\left[w_1\ \ \theta_1\ \ w_2\ \ \theta_2\right]^T \tag{8}$$

Following the Rayleigh-Ritz procedure again, the plate stability equation obtained, which after expanding it becomes $[K]\{\widetilde{W}\}-[B]\{\widetilde{W}\}=0$, in that $\{\widetilde{W}\}$ is the generalized vector of nodal displacements, and $[K]$, $[B]$ are the global elastic stiffness and geometric stiffness matrices respectively, which are determined by:

$$[K]=\int_{r_i}^{r_o}\left[\frac{d^2\Phi(r)}{dr^2}\quad \frac{1}{r}\frac{d\Phi(r)}{dr}\right]\begin{bmatrix}D & \nu D\\ \nu D & D\end{bmatrix}\begin{Bmatrix}\dfrac{d^2\Phi^T(r)}{dr^2}\\ \dfrac{1}{r}\dfrac{d\Phi^T(r)}{dr}\end{Bmatrix}r\,dr\,,$$

$$[B]=2\int_{r_i}^{r_o}N_r\left[\frac{d\Phi(r)}{dr}\right]\left\{\frac{d\Phi^T(r)}{dr}\right\}r\,dr \tag{9}$$

where, N_r has been previously found as $N_r=T_0\overline{N}(r)$. Substituting this equation in the second of Eqs. (9), the geometric stiffness may be rewritten as $[B]=T_0[B']$. So, the finite element matrix equation takes the form:

$$[B']^{-1}[K]\{\widetilde{W}\}=T_0\{\widetilde{W}\} \tag{10}$$

The recent equation is an eigenvalue problem, which can be solved to obtain the critical values of the temperature index T_{0cr} as the eigenvalues, and the mode shapes of buckling as the eigenvectors.

Numerical Results and Discussion

Thermal buckling of homogenous circular plates with linear variations in their thickness, and simply supported at their edges are studied. The numerical values of the thermal buckling load factor $\lambda_T=12(1+\nu)T_{0cr}(r_0/h)^2$ obtained from the present FEM, along with those reported in Refs. [1], [4] are summarized in Table 1., sorted for 9 different values of the taper parameter $\beta=(h_i-h_o)/h_i$, and 4 different values of h_i/r_o. there is good agreement between the results of the present FEM with those reported in the references, particularly those associated with the smaller values of h_i/r_0 ratio. Also, the critical buckling temperature index, T_{0cr}, is computed

Table 1. Comparisons of λ_T for linearly tapered plate.

h_i/r_0	Ref.	β=0.4	0.2	0.1	0.0	-0.1	-0.2	-0.4
.001	Present	8.3154	11.3569	12.9848	14.6819	16.4468	18.2776	22.1318
	[1]	8.3160	11.3578	12.9855	14.6825	16.4472	18.2780	22.1319
	[4]	8.2517	11.3572	12.9829	14.6819	16.4574	18.3133	22.2276
0.05	Present	8.3154	11.3569	12.9848	14.6819	16.4468	18.2776	22.1318
	[1]	8.2660	11.2660	12.8660	14.5299	16.2557	18.0412	21.7833
	[4]	8.3020	11.2657	12.8636	14.5296	16.2661	18.0766	21.8787
0.10	Present	8.3154	11.3569	12.9848	14.6819	16.4468	18.2776	22.1318
	[1]	8.1200	10.9999	12.5208	14.0911	15.7076	17.3669	20.8010
	[4]	8.1562	11.0001	12.5186	14.0909	15.7178	17.4013	20.8944
0.15	Present	8.3154	11.3569	12.9848	14.6819	16.4468	18.2776	22.1318
	[1]	7.8878	10.5835	11.9849	13.4159	14.8719	16.3485	19.3467
	[4]	7.9241	10.5841	11.9830	13.4157	14.8818	16.3813	19.4368

for uniform thickness annular plates of h/r_o=0.2, and r_i/r_o=2, under a uniform thermal load ($T(r)=T_0$) and plotted v.s. the volume fraction index, k, as shown in Fig. 3. The results are plotted for two different kinds of supporting, a clamped at inner and outer edges (C-C) and a simply supported at both edges (S-S). The value of T_{0cr} decreases as k increases for the two supporting kinds.

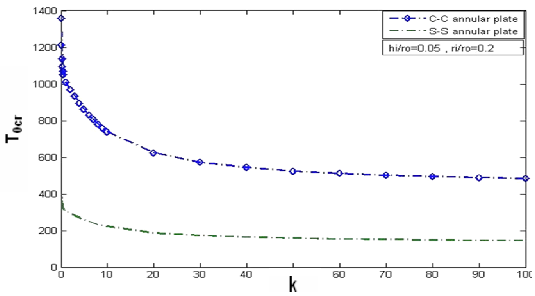


Fig. 3. Effect of volume fraction index, k, on T_{0cr} .

Concluding Remarks

A finite element formulation is derived for the thermal buckling behavior of axisymmetric FGM annular plates of variable thickness, subjected to a temperature rise distributed non-uniformly along radial coordinate axis, and validated through comparison with references. Parametric studies show that T_{0cr} decreases as the volume fraction index, k, increases. Clamped plates have larger T_{0cr} than their simply supported counterparts. In linearly tapered plates T_{0cr} decreases as β increments.

References

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