The Thermal Problem of Functionally Graded Material Substrate-coating Structure

Lifang Guo[1] Xing Li[2]

the depart of mathematic of Shanghaijiaotong university, Shanghai, China [1]

the college of mathematic of Ningxia university, YingChuan, China[2]

Introduction

As an alternative to the conventional homogeneous coatings, the functionally graded material coatings (FGM coatings) can effectively reduce the mismatch of material properties at the coating-substrate interface. Experimental results [1, 2] manifest that the resistance of the functionally graded coatings to thermal fatigue was approximately five times better than that of the duplex coatings. Results also show that the graded coating was superior to the duplex coating in long-term oxidation resistance.

In designing components involving FGMs, an important aspect of the problem is the fracture failure. Numerous studies have been performed to analyze this problem. In this paper, the thermal problem of the functionally graded coating-substrate structure is considered. The FGM is modeled as nonhomogeneous isotropic elastic medium. The problem is solved under the assumption of plane elasticity. The equations of heat conduction and elasticity are converted analytically into singular integral equations which are solved numerically to yield the crack tip stress intensity factors under thermo-mechanical loading. The main objective of the paper is to study the effect of the material nonhomogeneity parameters, partial crack surface insulation and the thickness of each layer on the crack tip stress intensity factors for the purpose of gaining better understanding of the thermo-mechanical behavior of graded materials.

Description of the problem

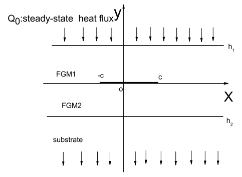

Figure 1: the considered problem

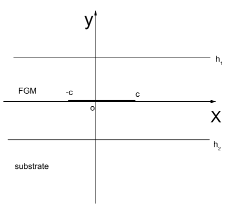

Figure 2: problem a

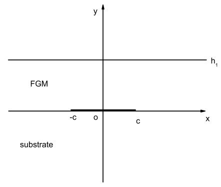

Figure 3: problem b

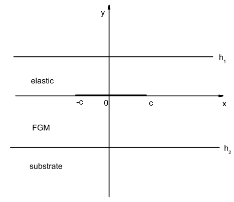

Figure 4: problem c

As shown in Figure 1, the problem under consideration consists of two infinitely long functionally graded strips of thickness h_1 and h_2 bonded to a homogeneous semi-infinite medium. There is an partially insulated crack of length $2c$ along the x-axis. The material gradient is oriented along the y-direction. The Poissons ratio υ is assumed to be a constant because the effect of its variation on the crack-tip stress intensity factors was shown to be negligible and is equal to the same value as that of the homogeneous substrate. The remaining thermo-mechanical properties depend on the y-coordinate only and are modeled by an exponential function as follows: $k_i(y) = k_0 e^{\delta_i y}$, $\mu_i(y) = \mu_0 e^{\beta_i y}$, $\alpha_i(y) = \alpha_0 e^{\gamma_i y}$, $k_3 = k_0 e^{-\delta_2 h_2}$, $\mu_3 = \mu_0 e^{-\beta_2 h_2}$, $\alpha_3 = \alpha_0 e^{-\gamma_2 h_2}$, where $i = 1, 2$, k_i, μ_i and α_i are the heat conductivity, the shear modulus and the thermal expansion coefficient respectively, k_0, μ_0 and α_0 correspond to the values of k_i, μ_i and α_i along the crack plane, α β and γ are the nonhomogeneity parameters controlling the material property variation. The loading consists of a steady-state heat flux Q_0 applied in y-direction away from the crack region. This model is reasonable because of the two factors : one is that inter-diffusion of molecules or atoms between the two different constituents might occur due to high temperature and pressure during the fabrication process of most bi-material composites, the materials within some extent of both sides of interface might be change as bi-FGm one[3]; the other is that it can study most of the problem about the coating-substrate structure, for example, the internal crack of the coating-substrate if $\delta_1 = \delta_2$ and $h_2 \neq 0$ (figure 2: problem a) and the interface crack if $h_2 = 0$ (figure 3: problem b) or the coating-intermediate layer-substrate if $\delta_1 = 0$, $\delta_2 \neq 0$ and $h_2 \neq 0$ (figure 4: problem c).

For the temperature field the heat equation is subject to the following boundary conditions:

$$k_1 \frac{\partial T_1(x,h_1)}{\partial y} = -Q_0, \quad y = h_1, \quad |x| < +\infty \quad (1)$$

$$k_3 \frac{\partial T_3(x,y)}{\partial y} = -Q_0, \quad y \to -\infty, \quad |x| < +\infty \quad (2)$$

$$-k_1(0) \frac{\partial T_1(x,0^+)}{\partial y} = \frac{1}{R_c}\left[\, T_1(x,0^+) - T_2(x,0^-) \,\right] \quad |x| < c \quad (3)$$

$$T_1(x,0^+) = T_2(x,0^-), \quad |x| > c, \qquad \frac{\partial T_1(x,0^+)}{\partial y} = \frac{\partial T_2(x,0^-)}{\partial y}, |x| < \infty \quad (4)$$

$$T_2(x,-h_2^+) = T_3(x,-h_2^-), \quad |x| > c, \qquad \frac{\partial T_2(x,-h_2^+)}{\partial y} = \frac{\partial T_3(x,-h_2^-)}{\partial y}, \quad |x| < \infty \quad (5)$$

The equation (3) describes the partial insulation of the crack surfaces which is modeled by assuming that the crack allows some heat flux Q_y that is only a certain percentage of the flux Q_0 corresponding to the perfect conduction case, R_c is the heat conductivity index assumed to be a constant, the limiting values 0 and 1 represent, respectively, perfect insulation and perfect conduction along the crack surfaces. For the displacement field the plane elasticity equations are subject to the following boundary conditions:

$$\sigma_{1xy}(x,0^+) = \sigma_{2xy}(x,0^-) = 0, \qquad \sigma_{1yy}(x,0^+) = \sigma_{2yy}(x,0^-) = 0, \quad |x| \leq c \quad (6)$$

$$\sigma_{2xy}(x,-h_2) = \sigma_{2yy}(x,-h_2) = 0, \qquad \sigma_{1xy}(x,h_1) = \sigma_{1yy}(x,h_1) = 0, \quad |x| \leq \infty \quad (7)$$

$$\sigma_{1xy}(x,0^+) = \sigma_{2xy}(x,0^-) \qquad \sigma_{1yy}(x,0^+) = \sigma_{2yy}(x,0^-), \quad |x| \geq c \quad (8)$$

$$u_1(x,0^+) = u_2(x,0^-), \qquad v_1 = (x,0^+) = v_2(x,0^-), \quad |x| \geq c \quad (9)$$

A set of coupled singular integral equations of Cauchy type can be get by using the integral transform and the heat equation, the plane elasticity equations and the related boundary conditions, the detailed solving approaches can reference the paper[4]. The resulting equation is solved numerically by using orthogonal Gauss-Chebyshev polynomials to yield the Mode I stress intensity factors. The numeric studies for the considered problem (figure 1) indicate that : for the crack surface temperatures : (1) the temperature jump across the crack becomes more pronounced as the increase of the heat conduction parameter ratio δ_2/δ_1 and the heat conductivity index R_c; (2) at certain conditions, the change of the width of coating and intermediate layer has little effect on the the temperature jump across the crack; for the thermal crack surface stresses: (1) the effect of the stiffness parameter ratio β_2/β_1 on the normalized stress intensity factors is influenced by the heat conductivity index R_c; (2) the ratio of the thermal expansion has little infect on the normalized stress intensity factors; (3) the normalized stress intensity factors reduce as the increase of the width of coating and intermediate layer but as the the heat conductivity index R_c shrink, the influence become weak. The same reach are also carried for the degenerate problems (problem a, b, c), the main results of the research demonstrate the influence of heat conductivity index R_c can not be ignored when study the therm-mechanical problem of the FGM coating-substrate structure. To improve the efficient of the FGM coating-substrate structure, the method of adjusting the width of the FGM layer is not the good one, the main way is to optimize the design of the gradient of FGMs.

reference

[1] K.A. Khor , Y.W. Gu Thermal properties of plasma-sprayed functionally graded thermal barrier coatings Thin Solid Films 372 2000 104-113

[2] N. Araki, D. W. Tang, and A. Ohtani Evaluation of Thermophysical Properties of Functionally Graded Materials International Journal of Thermophysics (2006) DOI: 10.1007/s10765-006-0034-5

[3] Li,Y.D.,Lee,K.Y. An anti-plane crack perpendicular to the weak/micro-discontinuous interface in a bi-FGM structure with exponential and linear non-homogeneities ,International Journal of Fracture ,2007,146,203-211

[4] EI–Borgi, S., Erdogan, F., Hidri, L. A partially insulated embedded crack in an infinite functionally graded medium under thermal–mechanical loading. International Journal of Engineering Science. 2004, 42, 371–393