FREE VIBRATION ANALYSIS OF GRID STIFFENED COMPOSITE CYLINDRICAL SHELLS

Milad Hemmatnezhad[*] and Hossein Rahimi

[*] Department of Mechanical Engineering, Tarbiat Modares University, P. O. Box 14115-143 Tehran, Iran
E-mail: m.hemmatnezhad@modares.ac.ir

Abstract
In this study, a unified analytical approach is applied to investigate the vibrational behavior of grid stiffened composite cylindrical shells. A smeared method is employed to superimpose the stiffness contribution of the stiffeners with those of shell in order to obtain the equivalent stiffness parameters of the whole panel. Theoretical formulations are based upon Sanders' thin shell theory. The Influence of variations in shell geometrical parameters and changes of the cross stiffeners angle on the shell frequencies are studied.
Keywords: Vibration; Analytical approach; Grid stiffened cylindrical shells; Sanders' theory

Introduction

Cylindrical shells due to their high strength as well as light weight have gained widespread use in most branches of structural engineering such as in launch vehicles, spacecrafts and etc. Large number of publications which expanded rapidly in the past decades testifies to this. An excellent collection of research in this area was carried out by Leissa [1]. There are also some good reviews on vibration of composite shells using experimental, analytical and numerical techniques [2–4].
Grid stiffened cylinders are cylinders with stiffening structures either on the inner, outer or both sides of the shell. These stiffeners significantly increase the load resistance of a cylinder without much increase in weight. The promising future of stiffened composite cylinders has in turn led to an extensive research work [5, 6].
The number of publications deal with the mechanical behavior of composite cylinder with cross stiffeners is scarce. Kidane et al. [6] derived the buckling loads of a generally cross and horizontal grid stiffened composite cylinder by developing a smeared method for determination of the equivalent stiffness parameters of a grid stiffened composite cylindrical shell. Recently, Yazdani and Rahimi have performed investigations on the buckling behavior of composite cylindrical shells with cross stiffeners [7].
In the present work a calculation of overall response of simply supported orthotropic cylindrical shells with horizontal and cross stiffeners is presented using an exact analytical approach with theoretical formulations based upon Sanders' theory. The Influence of variations in shell geometrical parameters and changes of the cross stiffeners angle on the shell frequencies are studied.

Equivalent stiffness

Consider a composite cylindrical shell reinforced with an isogrid stiffener structure as shown in Fig. 1. First of all it is required to determine the equivalent stiffness parameters of the overall structure in order to calculate the vibration frequencies of a composite cylinder with inner stiffening structure. The analytical tool employed for this, so called the smeared stiffener approach, uses a mathematical model to smear the stiffeners into an equivalent laminate and determine the equivalent orthotropic stiffness of the laminate (For further details the reader is referred to [6]).

Fig. 1 Unit cell and coordinate system for an isogrid stiffened cylindrical shell.

The relationships between boundary forces and strains for a cylindrical shell are given as

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} V_s N^s + V_{sh} N^{sh} \\ V_s M^s + V_{sh} M^{sh} \end{bmatrix} = \begin{bmatrix} V_s A^s + V_{sh} A^{sh} & | & V_s B^s + V_{sh} B^{sh} \\ V_s B^s + V_{sh} B^{sh} & | & V_s D^s + V_{sh} D^{sh} \end{bmatrix} \begin{bmatrix} \varepsilon^o \\ \kappa \end{bmatrix} \quad (1)$$

where the s and sh superscripts stand for stiffener and shell respectively. In the above equation, V_s and V_{sh} are the volume fractions of stiffener and shell respectively. A, B and D define the extensional, coupling, and bending stiffness coefficients, respectively. ε and κ are the strains and curvatures defined as below based on the Sanders' thin shell theory

$$\varepsilon_x^0 = u_{,x}, \qquad \varepsilon_\theta^0 = \frac{1}{R}(v_{,\theta}), \qquad \varepsilon_{\theta x}^0 = v_{,x} + \frac{1}{R}u_{,\theta},$$

$$\kappa_x = -w_{,xx}, \kappa_\theta = -\frac{1}{R^2}(w_{,\theta\theta} - v_{,\theta}), k_{x\theta} = \frac{-2}{R}(w_{,x\theta} - \frac{3}{4}v_{,x} + \frac{1}{4R}u_{,\theta}) \quad (2)$$

Equations of Motion

The following shell equations according to Sanders' theory in terms of axial, x, and circumferential, θ, coordinates, are used

$$RN_{x,x} + N_{x\theta,\theta} = RI_1\ddot{u}$$

$$RN_{x\theta,x} + N_{\theta\theta,\theta} + \frac{1}{R}M_{\theta\theta,\theta} + M_{x\theta,x} = RI_1\ddot{v} \quad (3)$$

$$RM_{x,xx} + 2M_{x\theta,x\theta} + \frac{1}{R}M_{\theta,\theta\theta} - N_\theta = RI_1\ddot{w}$$

where u, v and w are the axial, tangential and radial displacements, respectively. In above equation, I_1 is the inertia term for composite shell. A comma before a subscript denotes differentiation with respect to that subscript and dots denote time derivatives.

Method of solution

For a circular cylindrical shell the displacement field can be written in the following form for any circumferential wave number n

$$u(x,\theta,t) = \left(A_{0n} + \sum_{m=1}^{\infty} A_{mn} \cos(m\pi x / L) \right) \cos n\theta \sin \omega t$$

$$v(x,\theta,t) = \left(\sum_{m=1}^{\infty} B_{mn} \sin(m\pi x / L) \right) \sin n\theta \sin \omega t \qquad (4)$$

$$w(x,\theta,t) = \left(\sum_{m=1}^{\infty} C_{mn} \sin(m\pi x / L) \right) \cos n\theta \sin \omega t$$

where ω is the natural frequency of the shell. This set fulfills the exact solution for the shell with simply supported ends with No Axial constraint (SNA-SNA) which has boundary conditions at each end of the form

$$N_x = v = w = M_x = 0 \qquad (x = 0, L) \qquad (5)$$

Substitution of the set of displacement functions and their derivatives into Eq. (3) leads to an explicit relation for A_{0n} and a matrix equation in which A_{mn}, B_{mn} and C_{mn} are coupled together

$$\left\{ \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ & K_{22} & K_{23} \\ \text{Symm.} & & K_{33} \end{bmatrix} - \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_1 \end{bmatrix} \omega^2 \right\} \begin{bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}, \qquad (6)$$

$$(K_{01} - I_1 \omega^2) A_{0n} = F_{01}$$

For a non-trivial solution of Eq. (6), the determinant of the coefficient matrix must vanish,

$$\begin{vmatrix} K_{11} - I_1 \omega^2 & K_{12} & K_{13} \\ & K_{22} - I_1 \omega^2 & K_{23} \\ \text{Symm.} & & K_{33} - I_1 \omega^2 \end{vmatrix} = 0 \qquad (7)$$

resulting in a characteristic equation whose eigenvalues are the natural frequencies of the SNA-SNA shell. The corresponding eigenvectors also determine the mode shapes.

Results and discussion

The type of composite material used hereafter is HS-Graphite/Epoxy. The geometrical parameters of the considered grid stiffened cylinder are as follows

$$L = 180 \, \text{mm}, R = 70 \, \text{mm}, b = 90 \, \text{mm}, A_s = 6 * 2.8 \, \text{mm}^2, \phi = 30°, \theta = 30°$$

Fig. 2 shows the variation of the fundamental natural frequency of an isogrid stiffened composite cylinder against the shell length. It can be seen that with an increment in the shell length, the fundamental natural frequency of both unstiffened and stiffened shells decrease and the difference between two is significant for lower values of the length. Also, the values of the frequencies for the unstiffened shell are higher than that of stiffened shell. This is mainly because of the grid structure which results into an increase in the mass and a decrease in the natural frequency as a consequence. The effect of the cross stiffeners angle on the fundamental natural frequency is examined in Fig. 3 for three values of the circumferential wave numbers which clarifies the significant effect of this angle on the vibration frequencies.

Fig. 2 Variation of the fundamental natural frequency with the shell length ($n = 1$).

Fig. 3 Variation of the fundamental natural frequency with the cross stiffeners angle.

Conclusion

A unified exact analsis is employed to investigate the dynamic behavior of grid composite circular cylindrical shells. The present approach can be generalized to obtain the vibration frequencies of shells with different boundary conditions. Results obtained clarify that the natural frequencies of stiffened cylinder are lower than that excluding the effect of stiffeners. Also, the cross stiffeners angle has significant effect on the vibration of the stiffened cylinder. Further, with an increment in the shell length, the fundamental natural frequency of both unstiffened and stiffened shells decrease.

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