DIVERGENCE OF COMPOSITE AIRCRAFT WINGS ACCOUNTING THE AERODYNAMIC FINITE-SPAN EFFECT

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Introduction

Due to their high efficiency and significant advantages, thin-walled composite structures have been increasingly used in aerovehicles. Compared with metallic materials, composite materials are anisotropic, and their directional property can be designed, which, in the context of aeroelasticity, has generated a new technology referred to as the aeroelastic tailoring. The behavior of composite thin-walled beams is more complex than metallic ones in the sense that it is influenced by a number of important nonclassical effects such as transverse shear, warping inhibition, and nonuniformity of shear stiffness. In many research work devoted to the static aeroelastic problem of the composite aircraft wings[1], the aerodynamic strip theory is usually used for simplicity. In this paper, an anisotropic thin-walled box beam is considered as the load-carrying structure, and the aerodynamic finite-span effect is also considered.

Equations of motion

A box cross-section beam and with the circumferentially asymmetric stiffness (CAS) lay-up is adopted [2].

$$\delta u_0 : \ a_{14}v_0'' + a_{44}(u_0'' + \theta_z') = 0$$

$$\delta v_0 : \ a_{11}v_0'' + a_{14}(u_0'' + \theta_z') + a_{15}(w_0'' + \theta_x') = 0$$

$$\delta w_0 : \ a_{15}v_0'' + a_{55}(w_0'' + \theta_x') + p_z = 0$$

$$\delta \phi : \ a_{27}\theta_z'' + a_{37}\theta_x'' + a_{77}\phi'' - a_{66}\phi^{(\mathrm{IV})} + m_y = 0$$

$$\delta \theta_x : \ a_{33}\theta_x'' + a_{37}\phi'' - a_{15}v_0' - a_{55}(w_0' + \theta_x) = 0$$

$$\delta \theta_z : \ a_{22}\theta_z'' + a_{27}\phi'' - a_{14}v_0' - a_{44}(u_0' + \theta_z) = 0$$

Green functions via state-space method

Denote the follow non-dimensional parameters:

$$\hat{y} = y/L \quad SR = L/b \quad (\hat{\ })' = (\hat{\ })/d\hat{\eta}$$

$$\hat{p}_z = L^3 p_z / a_{33} \quad \hat{m}_x = L^2 m_x / a_{33} \quad \hat{m}_y = L^2 m_y / a_{33}$$

$$\hat{u}_0(\hat{y}) = u_0(y)/2b \quad \hat{v}_0(\hat{y}) = v_0(y)/L \quad \hat{w}_0(\hat{y}) = w_0(y)/2b$$

$$\phi(\hat{y}) = \hat{\phi}(\hat{y}) \quad \theta_x(\hat{y}) = \hat{\theta}_x(\hat{y}) \quad \theta_z(\hat{y}) = \hat{\theta}_z(\hat{y})$$

The motion equations can be written as:

$$\hat{x}' = A\hat{x} + f$$

where

$$\hat{x} = \left[\hat{u}_0 \ \hat{u}_0' \ \hat{v}_0 \ \hat{v}_0' \ \hat{w}_0 \ \hat{w}_0' \ \hat{\phi} \ \hat{\phi}' \ \hat{\phi}'' \ \hat{\phi}''' \ \hat{\theta}_x \ \hat{\theta}_x' \ \hat{\theta}_z \ \hat{\theta}_z' \right]^T$$

and f is a vector about external force. Let $\hat{p}_z = \delta(\hat{y} - \hat{y}_0)$ $\hat{m}_x = \hat{m}_y = 0$, where

$$\delta(\hat{y} - \hat{y}_0) = \begin{cases} 1 & at\ \hat{y} = \hat{y}_0 \\ 0 & others \end{cases}$$

Use the boundary condition at $\hat{y} = 0$ and $\hat{y} = 1$ to solve the state-space equation, and we can get the green function $C^{\phi w}(y, y_0)$ from the seventh and sixth element of the vector $\hat{x}(\hat{y}, \hat{y}_0)$ respectively. Similarly, let $\hat{m}_y = \delta(\hat{y} - \hat{y}_0)$, $\hat{m}_x = \hat{p}_z = 0$, we can get the Green function $C^{\phi\phi}$. Used these green functions, the elastic deformation can be written as:

$$\Phi = \int_0^1 C^{\phi w} \hat{p}_z d\hat{\eta} + \int_0^1 C^{\phi\phi} \hat{m}_y d\hat{\eta}$$

Modeling of the divergence problem

For wings with rectangular planform, the Weissinger L-method[3] is adopted to calculate the spanwise load distribution. The formula is:

$$\alpha = \frac{1}{\pi} \int_{-1}^{1} \frac{dG}{d\hat{\eta}} \frac{d\hat{\eta}}{(\hat{y} - \hat{\eta})} + \frac{L}{\pi c} \int_{-1}^{1} \frac{dG}{d\hat{\eta}} \tilde{L}(\hat{y}, \hat{\eta}) d\hat{\eta}$$

Where $G = \Gamma/2LU$, $\tilde{L}(\hat{y}, \hat{\eta})$ is L-function, and α is the total angle of attack

$$\alpha = \Phi + \Phi_r$$

Here, Φ is due to the structure elastic deformation and Φ_r is due to other factor contribute to the total angle of attack.

Denote $\hat{y} = \cos\phi$, $\hat{\eta} = \cos\varphi$ and account for the finite span effect (the circulation is zero at wingtips), G can be expanded as a sine series.

$$G = \sum_{\mu=1}^{\infty} A_\mu \sin(\mu\varphi)$$

The divergence problem can be represented as:

$$\hat{q}E_{11} + E_{22} = 0 \quad \text{with} \quad \hat{q} = qL^4/a_{33}\,.$$

Numerical Results and Discussion

We use the material and structure parameters of a thin-walled composite beam listed in the Tables 1 and 2.

A nonswept wing with rectangular planform is adopted here. The thin-walled beam is put in the middle of the wing, the chord length $c = 4b$, the distance between the dynamic centre and the elastic axis $e = c/4$, and the length of the wing is equal to the length of the thin-walled beam.

Table 1. Data of the wing's material parameters

Material Parameter	Value
E_{11}	$2.068\times10^{11}\,N/m^2$
$E_{22} = E_{33}$	$5.17\times10^{9}\,N/m^2$
$G_{12} = G_{13}$	$2.55\times10^{9}\,N/m^2$
G_{23}	$3.10\times10^{9}\,N/m^2$
$\mu_{12} = \mu_{13} = \mu_{23}$	0.25

Table 2. Data of the wing's geometric parameters

Structure parameter	Value
Length L	$1.96m$
Width $2b$	$0.254m$
Depth $2d$	$0.06807m$
Wall Thickness h	$0.0102m$
Number of layers	6

Fig. 1 shows the vertical displacement w and the angle of twist ϕ along the span with ply angle $\theta = 80^\circ$ and $\theta = 100^\circ$ respectively, when the structure is subject to one $1N$ load at the tip. By solving the equations of the divergence problem, we get the relationship between the fiber ply angle and the dynamic pressure q under symmetric loads, as shown in Fig.2.

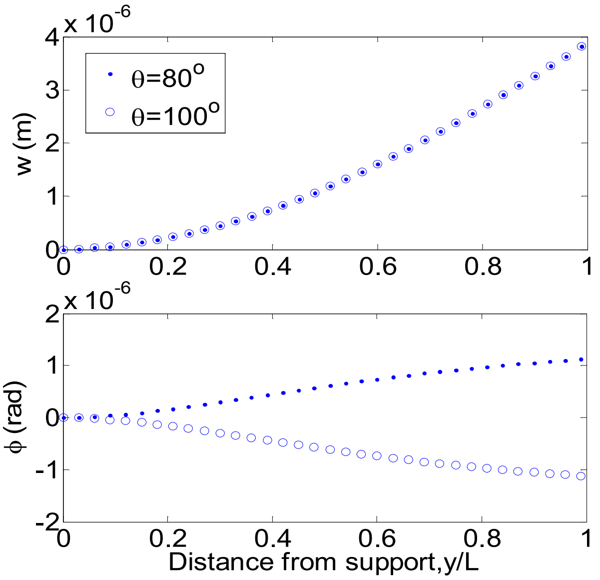

Fig.1 Distribution of the displacements subject to a $1N$ tip load.

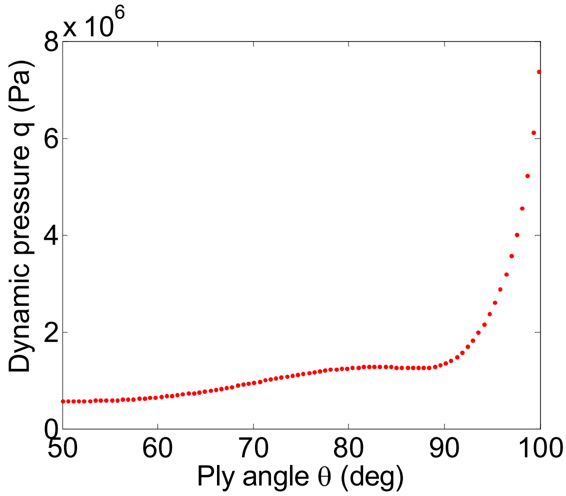

Fig.2 Dynamic pressure of divergence versus ply angle

Conclusions

Accounting for the aerodynamic finite-span effect, we investigate the divergence problem of an advanced aircraft wing modeled as a thin-walled composite beam. It is shown that the dynamic pressure of divergence changes rapidly when the ply angle crosses the point $\theta = 90^\circ$, because the upward bending deformation caused by the lift force increases the twist deformation (wash-in effect). However, when $\theta > 90^\circ$, the elastic twist reduces the angle of attack (wash-out effect), so the speed of divergence increases.

References

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