

Identifying material properties of composite materials from vibration data

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1. Introduction

Composite materials are engineered from two or more constituent materials with significantly different physical or chemical characteristics. They may be fabricated by combining two or more materials at macro-scale to obtain a useful structural material. Design of composite structures often requires an good understanding of their deflections and stress state under static and dynamic loads. Thus the determination of mechanical properties is one of the key issues in composite materials research. In view of the variety of composites available, the need for an efficient and reliable means of measuring or extracting the material properties has become even more important. The objective of this study is to develop an exact Fourier series method proposed by Khov et al. [1] into a hybrid technique for extracting the general material properties of an orthotropic plate. In particular, the method is used to identify four independent material constants for a symmetrically laminated thin plate.

2. Formulations

The equation of motion for a symmetrically laminated thin plate can be written as

$$D_{11} \frac{\partial^4 W}{\partial x^4} + 4D_{16} \frac{\partial^4 W}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^4 W}{\partial x^2 \partial y^2} + 4D_{26} \frac{\partial^4 W}{\partial x \partial y^3} + D_{22} \frac{\partial^4 W}{\partial y^4} - \rho h \omega^2 W(x, y) = q(\omega, x, y) \quad (1)$$

where ρ is the mass density of the plate, ω is the frequency of the harmonic motion, h is the plate thickness, and D_{ij} are the plate flexural stiffnesses from Classical Lamination Theory. As done in [1], the displacement function will be universally expressed as

$$W(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \cos(\lambda_{am}x) \cos(\lambda_{bn}y) + \sum_{l=1}^4 (\xi_b^l(y) \sum_{m=0}^{\infty} c_m^l \cos(\lambda_{am}x) + \xi_a^l(x) \sum_{n=0}^{\infty} d_n^l \cos(\lambda_{bn}y)) \quad (2)$$

By substituting Eq. (2) into governing equation and boundary conditions and equating the like terms on both sides, the final system of equations can be derived as

$$(\mathbf{K} - \rho h \omega^2 \mathbf{M}) \mathbf{A} = \mathbf{q} \quad (3)$$

The modal properties for an orthotropic plate can then be obtained from solving a standard matrix characteristic equation by setting $\mathbf{q}=\mathbf{0}$ in Eq. (3). It should be noted that the eigenvector \mathbf{A} for a given eigenvalue ω^2 contains the Fourier coefficients for the corresponding mode shape.

In the inverse solution process, we try to identify a set of model parameters (here, the elastic constants D_{ij}) so that the predicted natural frequencies can best match with the experimental results. This statement can be mathematically expressed in terms of an error norm to be minimized against the unknown parameters, that is,

$$\|\epsilon\| = \sum_{i=1}^M w_i (\lambda_i - \bar{\lambda}_i)^2 \quad (4)$$

where λ_i are the calculated natural frequencies from the method described earlier, $\bar{\lambda}_i$ are the measured natural frequencies from modal testing, and w_i are the weighting factors.

Minimization of Eq. (4) will lead to a set of nonlinear equations whose solutions are often solved using the multivariate Newton-Raphson method. The Newton's method is essentially an iterative technique which seeks a correction vector, $\mathbf{x} = \{\Delta x_1, \Delta x_2, \dots, \Delta x_N\}^T$, based on the error at the previous step:

$$\Delta \mathbf{x} = -(\mathbf{S}^T \mathbf{W} \mathbf{S})^{-1} \mathbf{S}^T \mathbf{W} \boldsymbol{\epsilon} \quad (5)$$

where

$$\mathbf{S}_{i,j} = \left\{ \frac{\partial \lambda_i}{\partial x_j} \right\} \quad (6)$$

$$\mathbf{W} = \text{diag}\{w_1 \ w_1 \ \dots\} \quad (7)$$

and

$$\boldsymbol{\epsilon} = \{\lambda_1 - \bar{\lambda}_1, \lambda_2 - \bar{\lambda}_2, \dots, \lambda_M - \bar{\lambda}_M\}^T \quad (8)$$

To generate the sensitivity matrix \mathbf{S} , it is necessary to explicitly calculate the derivatives of the (calculated) natural frequencies with respect to each of the model variables x_j . This can be easily done here since both the stiffness and mass matrices are explicitly defined as the functions of the key model parameters.

3. Results and Discussion

To verify the reliability and applicability of the proposed technique, an identification problem previously investigated by Deobald and Gibson

[2] will be reexamined. It involves extracting the elastic constants of a graphite epoxy plate using experimental natural frequencies measured under different boundary conditions. The related parameters for the plate are given as: $a=b=25.4$ cm, $h=1.688$ mm, $\rho=1.584$ g/cm³, $E_1=127.9$ GPa, $E_2=10.27$ GPa, $G_{12}=7.312$ GPa, $\nu_{12}=0.22$, and $\nu_{12}=0.0177$. Due to the space limit, only the results about the CFFF boundary condition (cantilever case) are reported here.

Table 1. Identified material properties for the graphite/epoxy plate.

Material Properties	Number of Natural Frequencies Utilized					
	4		5		6	
E_1 (GPa)	125.68	(-1.73 ⁺)	122.10	(-4.53)	122.411	(-4.29)
E_2 (GPa)	10.910	(6.24)	10.762	(4.79)	11.043	(7.53)
G_{12} (GPa)	6.7274	(-7.99)	6.6176	(-9.50)	6.7688	(-7.43)
ν_{12}	0.20738	(-5.74)	0.21069	(-4.23)	0.20508	(-6.78)

+ Relative error (%) as compared with the specified values.

In this study, the initial values for the elastic constants are “randomly” chosen as $0.98D_{11}$, $1.05D_{22}$, $1.1D_{12}$, and $0.9D_{66}$ by assuming that they are estimated reasonably well within an error of 10%. The identified parameters are presented in Table 1 for weighting factors $w_j=1/\bar{\omega}_j^2$ ($\bar{\omega}_j$ is the j -th measured natural frequency). It is seen that the elastic constants can be satisfactorily extracted as compared with the specified values. Varying weighting factors, the boundary conditions, and/or number of natural frequencies, allows creating redundancy of data for an improved consistency of the solutions. For instance, regardless of how a specimen is supported in a modal test, its elastic constants shall be unchanged and uniquely identified. Perhaps the best way to account for this fact is to average out the effect of the boundary conditions. It has been shown that such an averaging procedure can significantly reduce the error norm regardless of the weighting and mode selection schemes. As a matter of fact, the errors with the extracted parameters can be lowered to the same level of accuracy as that for the experimental modal data. This essentially says that averaging over different boundary conditions represents an

effective regularization of the identification process.

4. Conclusions

A hybrid method has been presented for determining the material properties of composite laminates based on a combined use of an analytical model and experimental modal data. Unlike in most numerical techniques, the sensitivity matrices can be explicitly derived from the analytical expressions for the stiffness and mass matrices. To improve the robustness of the inverse solution technique and the statistical significance of the results, different averaging schemes have been employed with respect to boundary conditions, numbers of natural frequencies, and/or weight factors.

- [1] Khov H, Li WL, Gibson RF. An accurate solution method for the static and dynamic deflections of orthotropic plates with general boundary conditions. *Composite Structures* 2009; 90: 474-481.
- [2] Deobald LR, Gibson RF. Determination of elastic constants of orthotropic plates by a modal analysis/Rayleigh-Ritz technique. *Journal of Sound and Vibration* 1988; 124(2): 269-283.