NATURAL FREQUENCY ANALYSIS OF THICK RECTANGULAR FGM PLATES UNDER IN-PLANE LOADING BY DIFFERENTIAL QUADRATURE METHOD

Saeed Jafari, Mansour Mohieddin Ghomshei

Department of Mechanical Engineering, Islamic Azad University- Karaj Branch, Karaj, Alborz, 31485-313, IRAN
Email: ms.saeedjafari@yahoo.com

Introduction

Functionally graded materials (FGMs) made of metal and ceramic, possess two main properties: toughness and a high degree of temperature-resistance. These special characteristics make them preferable in comparison to conventional laminates, which are subject to delamination, for some special engineering applications. The subject of natural frequency analysis of homogeneous rectangular plates has been studied by several researchers in recent years using different theories and methods (see Refs. [1],[2],[3]). The present article develops an analysis for the transverse vibration natural frequencies of a thick rectangular FGM plate under in-plane loading according to a higher order shear deformation theory (HSDT) using the generalized differential quadrature method (GDQM). Validation of the present numerical formulation is investigated, and effect of in-plane loading on the plate natural frequencies is studied.

Governing Equations

Consider a rectangular functionally graded plate (FGP) of length a, width b, and thickness h. The FGP assumed to be isotropic with material properties, such as Young's modulus, E, graded in the thickness direction according to a simple power-law in terms of the plate thickness coordinate z, and have symmetry with respect to the plate midplane., i.e.:

$$E^{(1)}(z) = E_m + (E_c - E_m)(-2z/h)^k, \quad -h/2 \le z \le 0, \qquad E^{(2)}(z) = E_m + (E_c - E_m)(2z/h)^k, \quad 0 \le z \le h/2 \tag{1}$$

where, subscripts m and c refer to the metal and ceramic constituents, respectively, and k is called as the volume fraction index. Poisson's ratio however assumed to be constant, as its variations is negligible.

Regarding the thick plate assumption, in the present study the third order shear deformation theory (HSDT) is implemented, which is based on the following displacement field:

$$u = u_0 + z\varphi_x - \frac{4}{3h^2}\left(\varphi_x + \frac{\partial w_0}{\partial x}\right)z^3, \quad v = v_0 + z\varphi_y - \frac{4}{3h^2}\left(\varphi_y + \frac{\partial w_0}{\partial y}\right)z^3, \quad w = w_0 \tag{2}$$

where, u, v, w are displacement components along x, y and z axes respectively, u_0, v_0, w_0 are midplane displacements, and φ_x, φ_y are pure rotations about x and y axes respectively. The xy plane of the coordinate system is located in the midplane of the plate. Regarding the large deflection occurrence due to inplane loading, the nonlinear von-Karman strain field is considered, hence the strain components in terms of displacements become:

$$\{\varepsilon\} = \{\varepsilon^{(0)}\} + z\{\varepsilon^{(1)}\} + z^2\{\varepsilon^{(2)}\} + z^3\{\varepsilon^{(3)}\} \tag{3}$$

where:

$$\{\varepsilon^{(0)}\} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}(\frac{\partial w_0}{\partial x})^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}(\frac{\partial w_0}{\partial y})^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{1}{2}(\frac{\partial w_0}{\partial x})(\frac{\partial w_0}{\partial y}) \\ \varphi_x + \frac{\partial w_0}{\partial x} \\ \varphi_y + \frac{\partial w_0}{\partial y} \end{Bmatrix}, \qquad \{\varepsilon^{(1)}\} = \begin{Bmatrix} \frac{\partial \varphi_x}{\partial x} \\ \frac{\partial \varphi_y}{\partial y} \\ \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \\ 0 \\ 0 \end{Bmatrix},$$

$$\{\varepsilon^{(2)}\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -\frac{4}{h^2}(\varphi_x + \frac{\partial w_0}{\partial x}) \\ -\frac{4}{h^2}(\varphi_y + \frac{\partial w_0}{\partial y}) \end{Bmatrix}, \qquad \{\varepsilon^{(3)}\} = \begin{Bmatrix} -\frac{4}{3h^2}(\frac{\partial \varphi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}) \\ -\frac{4}{3h^2}(\frac{\partial \varphi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}) \\ -\frac{4}{3h^2}(\frac{\partial \varphi_x}{\partial y} + 2\frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial \varphi_y}{\partial x}) \\ 0 \\ 0 \end{Bmatrix} \tag{4}$$

The stress-strain relations for the plate, as an isotropic laminae, can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ & Q_{11} & 0 & 0 & 0 \\ & & Q_{22} & 0 & 0 \\ & SYMM & & Q_{22} & 0 \\ & & & & Q_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{yz} \end{Bmatrix} \tag{5}$$

where:

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \qquad Q_{12} = \nu Q_{11} \tag{6}$$

Now, implementing Hamilton's principle, in that the stress and strain components are stated in terms of the derivatives of displacements by using Eqs. (3)-(5), a set of five differential equations governing the plate motion are derived. For the sake of brevity, only the first of the equations is depicted at the following:

$$\begin{aligned}&A_{11}\,\partial^2 u_0/\partial x^2+(A_{12}+A_{22})\,\partial^2 v_0/\partial x\partial y-[B_{11}+(4/3h^2)D_{11}]\partial^2\varphi_x/\partial x^2+\\&[B_{12}+B_{22}-(4/3h^2)(D_{12}+D_{22})]\partial^2\varphi_y/\partial x\partial y-(4/3h^2)D_{11}\,\partial^3 w_0/\partial x^3-\\&[(4/3h^2)(D_{12}+2D_{22})]\partial^3 w_0/\partial x\partial y^2+A_{22}\,\partial^2 u_0/\partial y^2+[B_{22}-(4/3h^2)D_{22}]\times\\&\partial^2\varphi_x/\partial y^2=I_0\,\partial^2 u_0/\partial t^2+[I_1-(4/3h^2)I_3]\partial^2\varphi_x/\partial t^2-(4/3h^2)I_3\,\partial^3 w_0/\partial t^2\partial x\\&\vdots\end{aligned}\tag{7}$$

where $A_{ij},B_{ij},C_{ij},D_{ij},F_{ij},H_{ij}$ are the plate linear and higher order stiffness coefficients, and I_1, I_2, …, I_6 are the plate inertias, which are defined by:

$$(A_{ij},B_{ij},C_{ij},D_{ij},F_{ij},H_{ij})=\int_{-h/2}^{h/2}(1,z,z^2,z^3,z^4,z^6)Q_{ij}dz,\quad i,j=1,2,$$

$$(I_0,I_1,I_2,I_3,I_4,I_6)=\int_{-h/2}^{h/2}(1,z,z^2,z^3,z^4,z^6)\rho(z)\,dz\tag{8}$$

Now, to solve simultaneously the set of motion equations, we implement the generalized differential quadrature method (GDQM). According to this numerical approximation method, a multiple derivative of a function such as $u_0(x,y)$ at an intermediate discrete point $(x_i\,,y_j)$ can be approximated by the weighted linear sum of the function values as [4]:

$$\frac{\partial^{r+s}u_0(x_i,y_j)}{\partial x^r\partial y^s}=\sum_{k=1}^{n_x}C_{ik}^{x(r)}\sum_{l=1}^{n_y}C_{jl}^{y(s)}u_0(x_k,y_l),$$

$$i=1,\dots,n_x\,,j=1,\dots,n_y\tag{9}$$

where $C_{ik}^{x(r)}$ and $C_{ik}^{x(r)}$ are entries of the weighting coefficient matrices of partial derivatives $\partial^r u_0/\partial x^r$, $\partial^s u_0/\partial y^s$ respectively. The equations of motion and the plate boundary conditions are descretized using GDQM, which led to a set of linear algebraic equations as an eigenvalue problem, that can be solved to obtain natural frequencies and mode shapes of vibration as the eigenvalues and eigenvectors respectively:

$$([K]-f^2[M])\{U\}=0\tag{10}$$

Numerical Results and Discussion

The first ten non-dimensional natural frequencies of a simply supported rectangular isotropic plate of a/b=2, and h/b=0.5, obtained by the present analysis along with an approximate numerical solution presented in Ref. [1] and an exact solution are exhibited in Table 1. The nondimensional frequency is defined by $\bar{f}=f\,h\sqrt{\rho_m/E_m}$. As can be seen in this Table, there is excellent agreement between the present results with the two other solution results. Furthermore, results of the present solution are even much closer to the exact solution results than those of Ref. [1]. In Fig. 1. values of non-dimensional fundamental frequency obtained for a FGM simply supported plate, is plotted versus nondimensional inplane loading N_{0xx}/F_{cr} applied to the plate, where F_{cr} is the plate critical buckling load. The plate is made of a mixture of Aluminum and Zirconia having properties $E_m=70\,GPa$, $\rho_m=2702\ \ kg/m^3$ and $E_c=151$ GPa, $\rho_c=3000kg/m^3$ respectively, and ν=0.3. The results are presented for 4 different values of volume fraction index k. The non-dimensional frequency increases as the tensile loading increases. Note that, negative values of N_{0xx}/F_{cr} which refer to the compressive axial forces, result in a decrease in the frequency, such that as N_{0xx}/F_{cr} approaches to -1, the frequency approaches to zero. Also, as the volume fraction index k increases, the plate fundamental frequency decreases.

Table 1. Comparison of the first ten natural frequencies of an isotropic thick plate.

Mode of Vibration	Non-dimensional Frequency		
	Ref. [1]	Present	Exact*
1	0.7857	0.7854	0.7854
2	1.0725	1.0731	1.0692
3	1.5248	1.5158	1.5158
4	1.5715	1.5708	1.5708
5	1.5763	1.5708	1.5708
6	1.7587	1.7562	1.7562
7	2.1500	2.1168	2.1168
8	2.2343	2.2214	2.2214
9	2.3766	2.3562	2.3562
10	2.5490	2.5235	2.5235

*The exact solution is also reported in Ref. [1].

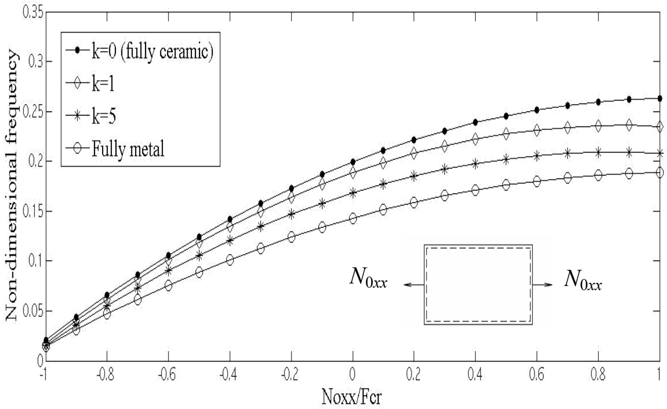

Fig. 1. Effect of inplane force N_{0xx} on the fundamental frequency of FGM thick plate.

Refrences

1. Batra R. C., Immaee S. A., Vibrations of thick isotropic plates with higher order shear and normal deformable plate theories, *Computers and Structures*, 78 (2007) 433–439.

2. Biancolini M. E., et al., Approximate solution for free vibrations of thin orthotropic rectangular plates, *Journal of sound and vibration*, 288 (2005) 321-344.

3. Leissa A. W., Kang J. H., Exact solution s for vibration and buckling of an SS-C-SS-C rectangular plate loaded by linearly varying in-plane stresses, *International journal of mechanical sciences*, 44 (2002) 1925-1945.

4. Bert C. W., Malik M., Differential Quadrature Method in Computational Mechanics: A Review, *Appl. Mech. Rev.* 49(1) (1996) 1-25.