# The Problem of the plane wave scatting by Substrate-coating Structure

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## Introduction

In most of the structural materials, crack or inclusion are present either as natural defects or as a result of fabrication process. These problems in elastodynamics are much important in view of their applications in geophysics and earthquake engineering. Many problems have been solved involving one or more cracks in an infinite homogeneous elastic medium. But the diffraction of elastic waves become more practical when boundaries are present in the medium. In this paper, it has been analyzed the diffraction of plane wave ( P wave and SV wave ) by a crack situated at the interface of the bonded elastic-strip and functionally graded strip. Using integral transform, the mixed boundary value problem can be reduced to the solution of a set of integral equations. Solving the integral equation numerically, the normalized dynamic stress intensity factors have been calculated and illustrated graphically to show the effects of the frequency of the incident wave, the type of the wave, the direction of the wave, the material gradient parameter of functionally graded coating and the thickness of the coating on the normalized dynamic stress intensity factors.

## Description of the problem

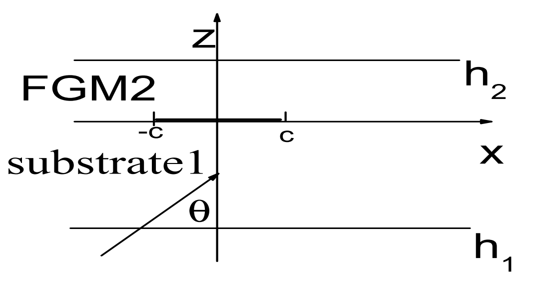


Fig.1. the problem of the plane wave diffraction by the crack of coating-substrate structure

The layered graded composite structure and the corresponding coordinate system are illustrated in Fig.1. It is assumed that the coating is in the region $0 \leq z \leq h_2$ and the substrate $-h_1 \leq z \leq 0$. The substrate is a homogenous isotropic material. The coating is a graded material and the material parameters vary smoothly in the thickness direction, which can be assumed as

$$\mu_2(z) = \mu_1 e^{\beta z}, \quad \rho_2(z) = \rho_1 e^{\beta z} \tag{1}$$

The equation of the constitute is

$$\begin{aligned} \sigma_{jxx} &= \frac{\mu_j}{\kappa - 1}\left[(1+\kappa)\frac{\partial u_j}{\partial x} + (3-\kappa)\frac{\partial \omega_j}{\partial z}\right], \\ \sigma_{jzz} &= \frac{\mu_j}{\kappa - 1}\left[(3-\kappa)\frac{\partial u_j}{\partial x} + (1+\kappa)\frac{\partial \omega_j}{\partial z}\right], \quad (\ j = 1,2\ ) \\ \sigma_{jxz} &= \mu_j\left[\frac{\partial u_j}{\partial z} + \frac{\partial \omega_j}{\partial x}\right] \end{aligned} \tag{2}$$

in the above equation ( 2 ) $\kappa = 3 - 4v$ for the plane strain and $\kappa = \frac{3-v}{1+v}$ for the plane stress.

The equilibrium equation is

$$\begin{aligned} \frac{\partial \sigma_{jxx}}{\partial x} + \frac{\partial \sigma_{jxz}}{\partial z} &= \rho_j \frac{\partial^2 u_j}{\partial t^2}, \\ \frac{\partial \sigma_{jxz}}{\partial x} + \frac{\partial \sigma_{jzz}}{\partial z} &= \rho_j \frac{\partial^2 \omega_j}{\partial t^2}, \quad (\ j = 1,2\ ) \end{aligned} \tag{3}$$

Using the superposition, the problem can be decomposed into two problems, one is the incident wave from the far field scattered by the same structure which no crack exists, this problem can be referred as incident problem, the other is the stress generated by the incident field on the crack in the presence of the same structure without incident from the far field, this problem can be referred as scattering problem.

For the incident problem the boundary condition is

$$\sigma_{1xz}^{in}(x, h_1) = 0, \quad \sigma_{1zz}^{in}(x, h_1) = 0 \tag{4}$$

$$\begin{aligned} u_2^{in}\left(x, 0^+\right) &= u_1^{in}\left(x, 0^-\right) + u^0\left(x, 0^-\right) e^{ikx} \\ \omega_2^{in}\left(x, 0^+\right) &= \omega_1^{in}\left(x, 0^-\right) + \omega^0\left(x, 0^-\right) e^{ikx} \\ \sigma_{2xz}^{in}\left(x, 0^+\right) &= \sigma_{1xz}^{in}\left(x, 0^-\right) + \sigma_{xz}^0\left(x, 0^-\right) e^{ikx} \\ \sigma_{2zz}^{in}\left(x, 0^+\right) &= \sigma_{1zz}^{in}\left(x, 0^-\right) + \sigma_{zz}^0\left(x, 0^-\right) e^{ikx} \end{aligned} \tag{5}$$

$$\sigma_{2xz}^{in}(x, h_2) = 0, \quad \sigma_{2zz}^{in}(x, h_2) = 0 \tag{6}$$

according to the boundary condition and referred to the paper [1] the stress field which the incident wave caused at the crack position can be found for the scattering problem.

For the the scattering problem, the boundary condition is

$$\sigma_{1xz}^{o}\left(x,h_1\right)=0,\qquad \sigma_{1zz}^{o}\left(x,h_1\right)=0 \tag{7}$$

$$\sigma_{2xz}^{o}\left(x,h_2\right)=0,\qquad \sigma_{2zz}^{o}\left(x,h_2\right)=0 \tag{8}$$

$$\begin{gathered} u_1^{o}\left(x,0^-\right)=u_2^{o}\left(x,0^+\right), \\ w_1^{o}\left(x,0^-\right)=w_2^{o}\left(x,0^+\right),\quad |x|>c \end{gathered} \tag{9}$$

$$\begin{gathered} \sigma_{1xz}^{o}\left(x,0^-\right)=\sigma_{2xz}^{o}\left(x,0^+\right), \\ \sigma_{1zz}^{o}\left(x,0^-\right)=\sigma_{2zz}^{o}\left(x,0^+\right),\quad |x|>c \end{gathered} \tag{10}$$

$$\begin{gathered} \sigma_{2xz}^{o}\left(x,0^+\right)=\sigma_{1xz}^{o}\left(x,0^-\right)=-\sigma_{2xz}^{i}, \\ \sigma_{2zz}^{o}\left(x,0^+\right)=\sigma_{1zz}^{o}\left(x,0^-\right)=-\sigma_{2zz}^{i} \end{gathered} \tag{11}$$

according to the boundary condition and taking advantage of the integral transform and auxiliary function, a set of coupled singular integral equations of Cauchy type can be get from which the express of the dynamic stress intensity factors can be get.

## Conclusion

The numeric studies considered the effects of the frequency of the incident wave, the type of the wave, the direction of the wave, the material gradient parameter of functionally graded coating and the thickness of the coating on the normalized dynamic stress intensity factors. For the P wave, the normalized stress intensity factor $K_I/K_{I0}$ increase with the greater gradient parameter and the bigger angle of incident wave wile the normalized stress intensity factor $K_{II}/K_{II0}$'s change is opposite. All the normalized stress intensity factors $K_I/K_{I0}$ and $K_{II}/K_{II0}$ decrease with the increase of the coating thickness parameter $h_2c$, but when the coating thickness parameter $h_2c > 1.0$ this change is not obvious. For the SV wave, the normalized stress intensity factor $K_I/K_{I0}$ decreases with the greater gradient parameter $\beta c$ wile the normalized stress intensity factor $K_{II}/K_{II0}$'s change is different. All the normalized stress intensity factors $K_I/K_{I0}$ and $K_{II}/K_{II0}$ decrease with the increase of the angle of SV wave but as the angle of incident wave increase this change is not distinct. The normalized stress intensity factor $K_I/K_{I0}$ increase with the bigger of the coating thickness parameter $h_2c$ and the change of $K_{II}/K_{II0}$ is different, but as same as the P wave, when the coating thickness parameter $h_2c$ big enough, this change is little. So the conclusion is that the gradient of the functionally graded material have crucial influence on the dynamic intensity factors, while the angle of the incident wave and the thickness of the coating may have some influence on the dynamic intensity factors, but this influence is limited . So a good way to prevent the destroy of the coating-functionally graded structure is to optimize the gradient of the functionally graded material.

## reference

[1] Xia, C. H. and Ma, L. Dynamic Behavior of a Finite Crack in Functionally Graded Material Subjected to Plane Incident Time-harmonic Stress Wave. Composite Structure. 2007, 77, 10-17