Electrohydrodynamic Generalized Dispersion of Unsteady Convective Diffusion in Couple Stress Poorly Conducting Fluid Bounded by Porous Layer

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1. INTRODUCTION

The study of dispersion in a channel bounded by rigid boundaries or in an impermeable cylinder has been investigated using (i) Taylor model (1953) valid for large time, (ii) Aris model (1956) an improvement on Taylor's model (iii) Gill and Shankarasubramainan model valid for all time have been investigated in the literature. However in biomechanical problems like cartilages in synovial joints, endothelium in arteries and trachea (wind pipe) in the body involve permeable boundaries having B.J. slip condition between the clear fluid and the permeable boundary. Further, in cartilages, the hylauranic acid, nutrients and in arteries, natural R.B.C.'s will be freely suspended executing microrotation. The fluid with microrotation is called micro polar fluid (Erringon 1966) and particular case of micropolar fluid is a couple stress fluid when microrotation balances with natural vorticity. These biological problems involve couple stress poorly conducting fluid. Such a study of poorly conducting fluid in the presence of electric field is electrohydrodynamics (EHD). A study of the dispersion of fluid with rigid boundaries is very restrictive and hence a proper study should involve B.J. slip, couple stress parameter and electric field which has not been given any importance in the literature. The study of it using generalized dispersion model is the main objective of this paper. To achieve this objective, the basic equations, the corresponding boundary and initial conditions are given in section 2. The solution and generalized dispersion co-efficient are given in section 3.

2. MATHEMATICAL FORMULATION

Following Rudraiah et.al. 1986, the basic equations for the flow in dimensionless form are given by

$$\frac{d^4U}{d\eta^4} - a^2\frac{d^2U}{d\eta^2} = -Ka^2(B_1 - B_2 + B_2\alpha\eta) \qquad (2.1)$$

$$\frac{\partial\theta}{\partial\tau} + U^*\frac{\partial\theta}{\partial\varsigma} = \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial\varsigma^2} + \frac{\partial^2\theta}{\partial\eta^2} \qquad (2.2)$$

Boundary conditions on velocity are

$$\partial U/\partial\eta = \mp\alpha\,\sigma(u_b - u_p)\ at\,\eta = \pm 1 \qquad (2.3a)$$

$$\partial^2 u/\partial\eta^2 = 0\ at\,\eta = \pm 1 \qquad (2.3b)$$

Initial conditions on concentration are

$$\theta(0, X, \eta) = 1 \text{ for } |x| \le (1/2)\,x_s \qquad (2.4a)$$

$$\theta(0, X, \eta) = 0 \quad \text{for} \quad |x| > (1/2)\,x_s \qquad (2.4b)$$

$$[\partial\theta/\partial\eta](\tau, X, 1) = 0 = [\partial\theta/\partial\eta](\tau, X, -1) \qquad (2.4c)$$

$$\theta(\tau, \infty, \eta) = [\partial\theta/\partial Y](\tau, \infty, \eta) = 0 \qquad (2.4d)$$

3. GENERALIZED DISPERSION COEFFICIENT

The solution of equation (2.1) satisfying the conditions (2.3) is

$$U = \frac{-K(B_1 - B_2)}{2}\left[1 - \eta^2 - \frac{2}{a^2}\left(1 - \frac{\cosh a\eta}{\cosh a}\right) + A_0\right] - \frac{KB_2\alpha\sinh a\eta}{a^2\sinh a} + A_1 + A_2\eta + A_3\eta^2 \qquad (3.1)$$

The solution of equation (2.2) using (2.4) following Gill and Shankarasubramanian model(1970) is given by

$$K_2(\tau) = \frac{1}{Pe^2} + A - \sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\left(B + C + E\right) + F \qquad (3.2)$$

where, $K_2(\tau)$ represent the most dominant dispersion coefficient which is time dependent for convection and diffusion. $K_2(\tau)$ for pure convection (i.e. neglecting the diffusion term in equation 2.2), is

$$K_2(\tau) = \frac{\tau}{72A_6^2}\left(\begin{array}{l}\frac{36}{a^4\cosh^2 a} - \frac{(48a^2 + 108)}{a^5}\tanh a \\ -\frac{72}{a^6}\tanh^2 a + \frac{8}{5a^4}(a^4 + 90) + G\end{array}\right) \qquad (3.3)$$

For want of space the constants are omitted here but they are included in numerical evaluation.

The exact solution of equation (2.2) subject to the condition (2.4) is

$$2\theta_m(\zeta, \tau) = erf\left[\left(\frac{X_s}{2} + \zeta\right)\left(2\sqrt{T}\right)^{-1}\right] + erf\left[\left(\frac{X_s}{2} - \zeta\right)\left(2\sqrt{T}\right)^{-1}\right]$$

where

$$T = \int_0^\tau K_2(z)dz \text{ and } erf(x) = \frac{1}{\pi}\int_0^x e^{-z^2}dz \qquad (3.4)$$

4. RESULTS AND DISCUSSIONS

The equations governing the flow for a poorly conducting couple stress fluid are the partial differential equations 2.1 and 2.2. These equations are solved analytically. Using these solutions the dispersion coefficient $\mathbf{k_2}$ is determined following the analysis of Gill and Shankarasubramanian and is given by equation 3.2. This equation is numerically computed for different values of σ and for a=1 and a=20. The results are depicted graphically in Figure 1. It represents the effects of porous parameter σ and couple stress parameter a. It shows that $\mathbf{k_2}$ increases with an increase in porous parameter σ and decreases with increase in couple stress parameter a. This result is useful in understanding one of the causes for haemolysis, which in turn useful in the design of an artificial organ. Initially $\mathbf{k_2}$ increases gradually for the value of $\boldsymbol{\tau}$ = **0.5** and remains uniform for values $\boldsymbol{\tau}$ greater than **0.5**. In this figure, the result $\sigma \to \infty$ corresponds to those given by Rudraiah et al (1986) for impermeable boundaries.

The transport of major metabolisis (such as sugars and amino acids) is rather slow and convective transport plays a major role in accelerating them. Therefore, we have computed $\boldsymbol{\Theta_m}$ given by (3.4) and the results are depicted. The variation of mean concentration $\boldsymbol{\Theta_m}$ with axial distance x for a fixed $\boldsymbol{\tau}$ are plotted in figure (2). This figure reveal a marked variation of $\boldsymbol{\Theta_m}$ with time and as σ increases the peak value of mean concentration decreases, a result that is true for combined convection plus diffusion and pure convection.

Figure (3) represents the variation of mean concentration $\boldsymbol{\Theta_m}$ with τ for different values of σ and for a fixed a. In this figure we note that $\boldsymbol{\Theta_m}$ for convection and diffusion and $\boldsymbol{\Theta_m}$ for pure convection increase with a decrease in σ and for small values of $\boldsymbol{\tau}$, $\boldsymbol{\Theta_{mcd}}$ curve is above $\boldsymbol{\Theta_{mc}}$. In other words the porous parameter significantly influences $\boldsymbol{\Theta_m}$. This information is also useful in understanding the cause for haemolysis.

Apart from the above biomechanical applications of this study, certain general conclusions of a mathematical nature can also be made. They are

(i)The couple stress are operative only for small values of 'a' and the present results reduce to Newtonian fluids in the limit of $a \to \infty$.

(ii)Taylor's dispersion model forms a particular case of the generalized dispersion model for asymptotic values of τ. In other words, the generalized dispersion model reduces to Taylor's dispersion model.

(iii)Results of Rudraiah et al (1986) form a particular case of the present study, in the sense that for large values of σ the results of present study reduce to those of Rudraiah et al (1986).

References

1. Taylor, G.I. 'Dispersion of Soluble Matter in Solvent Flowing Slowly through a Tube', Proc. Roy. Soc., London, A 219, (1953), 186.

2.ARIS, R., 1956: On the dispersion of a solute in a fluid flowing through a tube, Proc. Roy. Soc. London, A 235, 67.

3. A.C. Eringen, Theory of Micropolar Fluids, J. mathMech.16 (1966), pp. 1-18.

4. Gill W. M. and ShankarSubramanian R. 'Exact Analysis of Unsteady Convective Diffusion', Proc. Roy. Soc., London, A 316, (1970), 341.

5. Rudraiah, N., Dulal Pal and Siddheshwar, P.G., 'Effect of Couple Stresses on the Unsteady Convective Diffusion in Fluid Flow through a Channel', Biorheology, A 23(1986), 349.

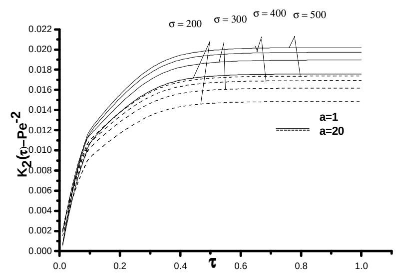

Figure 1: Effect of couplestress and porous parameter on unsteady dispersion coefficient

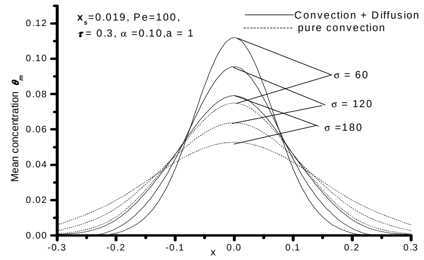

Figure 2: Comparision of of results of convective diffusion for the mean concentration distribution with that of pure convection for different values of σ

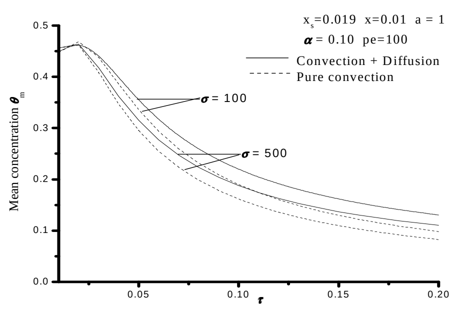

Figure 3: Plots of mean concentration $\boldsymbol{\Theta_m}$ verses τ