

## Enhanced optical transparency in 3D nanostructured films

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### INTRODUCTION

We present a very efficient recursive method to calculate the effective optical response of metamaterials made up of arbitrarily shaped inclusions arranged in periodic three-dimensional arrays. We consider dielectric particles embedded in a metal matrix with a lattice constant much smaller than the wavelength of the incident field, so that we may neglect the retardation and factor the geometrical properties from the properties of the materials. If the conducting phase is continuous the low frequency behavior is metallic. If the conducting paths are thin, the high frequency behavior is dielectric. Thus, extraordinary-transparency bands may develop at intermediate frequencies, whose properties may be tuned by geometrical manipulation.

Metallic films with sub-wavelength nanometric holes are known to display an extraordinarily large transmittance at near infrared frequencies for which the metal is opaque and light waves are not expected to propagate within the holes [1, 2]. The anomalous transmission at nano-structured metallic films opens the possibility of tailored design of the optical response for many applications that include hyperlens far-field-subdiffraction magnification [3–5], cloaking [6], optical antennas [7, 8], and circular polarizers [9].

In a recent works [10–12] we have developed a procedure to calculate the frequency-dependent macroscopic dielectric-response tensor  $\epsilon_{ij}^M$  of 2D and 3D-periodic lattices of inclusions with arbitrarily shaped cross sections. In this work we present Haydock’s recursive scheme [13] to obtain the macroscopic dielectric response of periodic metamaterials in the long wavelength limit. Our procedure yields a speed improvement of several orders of magnitude over that of Ref. [10]. This allows efficient calculations for three-dimensional structures with arbitrary geometry, including interpenetrated inclusions. We show that the geometry of the inclusions might lead to an enhanced anomalous transmission and a very anisotropic optical behavior. We found that enhanced transmittance is a generic phenomenon present within metal-dielectric metamaterials whenever there are only poor conducting paths across the whole sample.

### THEORY

We consider a metamaterial consisting of an artificial crystal made of an homogeneous material with a periodic lattice of arbitrarily shaped nanometric inclusions. We assume that each region  $\alpha = a, b$  is large enough though to have a well defined macroscopic response  $\epsilon_\alpha$  which we assume local and isotropic, but much smaller than the free wavelength  $\lambda_0 = 2\pi c/\omega$  with  $c$  the speed of light in vacuum and  $\omega$  the frequency. The microscopic response is described by

$$\epsilon(\mathbf{r}) = \epsilon_a - B(\mathbf{r})\epsilon_{ab} \quad (1)$$

where  $\epsilon_{ab} \equiv \epsilon_a - \epsilon_b$  and  $B(\mathbf{r})$  is the characteristic function for the  $b$  regions, which we assume periodic,  $B(\mathbf{r} + \mathbf{R}) = B(\mathbf{r})$ , with  $\{\mathbf{R}\}$  the Bravais lattice of the metamaterial. The Fourier transform of (1) is given by,  $\epsilon_{\mathbf{G}\mathbf{G}'} = \epsilon_a \delta_{\mathbf{G}\mathbf{G}'} - \epsilon_{ab} B_{\mathbf{G}\mathbf{G}'}$ , where  $B_{\mathbf{G}\mathbf{G}'} = (1/\Omega) \int_v d^3r e^{i(\mathbf{G}-\mathbf{G}')\cdot\mathbf{r}}$  describes the geometry of the inclusions which occupy the volume  $v$  within the unit cell of volume  $\Omega$ , where  $\mathbf{G}$  are the reciprocal lattice vectors. In particular,  $B_{\mathbf{00}} = v/\Omega \equiv f$  is the filling fraction of the inclusions. From Refs. 11, 12 the macroscopic (M) dielectric response defined through  $\mathbf{E}_M = \overleftrightarrow{\epsilon}_M^{-1} \cdot \mathbf{D}_M$  is given by

$$\overleftrightarrow{\epsilon}_M^{-1} = \hat{\mathbf{q}}\eta_{00}^{-1}\hat{\mathbf{q}}, \quad (2)$$

with

$$\eta_{\mathbf{G}\mathbf{G}'}^{-1} = \frac{1}{\epsilon_{ab}} [u(\omega)\delta_{\mathbf{G}\mathbf{G}'} - B_{\mathbf{G}\mathbf{G}'}]^{-1}, \quad (3)$$

where  $\hat{\mathbf{q}}$  are Cartesian directions and  $u(\omega) \equiv (1 - \epsilon_b(\omega)/\epsilon_a(\omega))^{-1}$  is a frequency dependent spectral variable. Eq. (3) can be efficiently solved by Haydock’s recursive scheme, resulting in [11, 12]

$$\eta_{00}^{-1} = \frac{u}{\epsilon_a} \frac{1}{u - a_0 - \frac{b_1^2}{u - a_1 - \frac{b_2^2}{u - a_2 - \frac{b_3^2}{\ddots}}}}, \quad (4)$$

where  $a_n$  and  $b_n$  are the Haydock’s coefficients, that depend only on the geometry through  $B_{\mathbf{G}\mathbf{G}'}$ . The dependence on composition and frequency is completely encoded in the complex valued spectral variable  $u$ . Thus, for a given geometry we may explore manifold compositions and frequencies without having to recalculate Haydock’s coefficients.

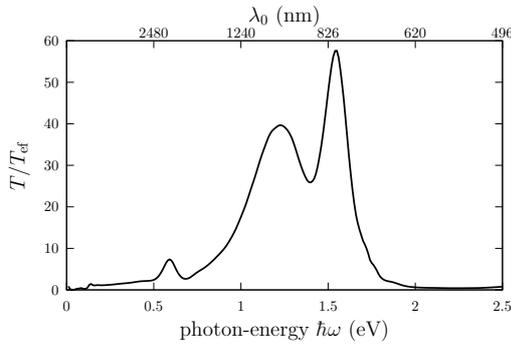


FIG. 1: Transmittance  $T$  vs. frequency  $\omega$  of a 200 nm film of an Au host with a simple cubic lattice of spheres of radius  $r = 0.6a$  with dielectric response  $\epsilon_b = 4$ , with  $a$  the lattice parameter, normalized to the transmittance  $T_{\text{eff}}$  of an effective Au film with the same amount of metal.

## RESULTS

In Fig. 1 we show the transmittance  $T$  of a film made of a simple cubic lattice of spherical dielectric inclusions with response  $\epsilon_b = 4$  within an Au host. We chose the radius as  $r = 0.6a$ , with  $a \ll \lambda_0$  the lattice parameter, so the spheres overlap their neighbors. We have normalized the results to the transmittance  $T_{\text{eff}}$  of an effective homogeneous Au film of width  $d_{\text{eff}}$ , in order to emphasize the transmittance enhancement due to the metamaterial geometry. Several enhancement peaks between one and two orders of magnitude are visible in the transmittance spectrum, corresponding to the excitation of coupled multipolar plasmon resonances within the region where the metal is opaque.

In Fig. 2 we show the normalized transmittance  $T_\alpha/T_{\text{eff}}$  ( $\alpha = x, y$ ) for plane polarized light normally incident on a 200 nm film lying on the  $xy$  plane made of a simple orthorhombic lattice of  $z$ -oriented dielectric cylinders with radius  $r = 0.53a_x$ , height  $h = 0.9a_z$  dielectric function  $\epsilon_b = 2$  and lattice parameters  $a_y = 1.15a_x$  and  $a_z = a_x$  within an Au host. There is a huge anisotropy, with a peak enhancement of  $T_y$  almost two orders of magnitude larger than that of  $T_x$ . For this geometry there is an overlap between neighbor cylinders along  $x$ , so the system is a better low frequency conductor along  $y$  and displays stronger finite-frequency dielectric-like resonances along  $x$ .

## CONCLUSIONS

In summary, we developed a formalism that allows very efficient calculations of the macroscopic response of 3D nano-structured periodic metamaterials. We applied it to films made of various lattices of dielectric inclusions with assorted shapes within opaque metallic hosts and found an extraordinary enhancement of the transmittance as a

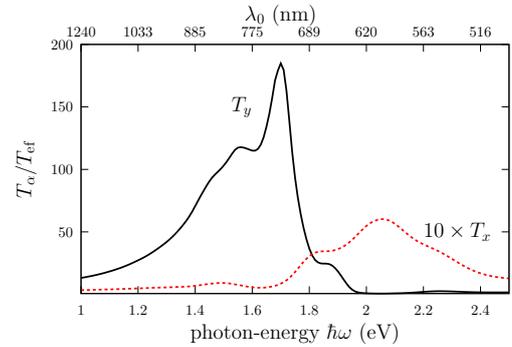


FIG. 2: (Color online) Transmittance  $T_\alpha$  vs. frequency  $\omega$  of a 200 nm film with faces normal to the  $z$  axis illuminated by light polarized along  $\alpha = x, y$ . The film consists of a simple orthorhombic lattice of  $z$ -oriented cylinders with radius  $r = 0.53a_x$ , height  $h = 0.9a_z$  and dielectric response  $\epsilon_b = 2$  with lattice parameters  $a_x = a_z \ll \lambda_0$  and  $a_y = 1.15a_x$  within an Au host.  $T_\alpha$  is normalized to the transmittance  $T_{\text{eff}}$  of an effective Au film.

generic property whenever the metamaterial is conducting at low frequencies and has dielectric like resonances at larger frequencies. It does not require holes that pierce the whole film on whose surface SPP's may propagate.

## Acknowledgment

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