NONLINEAR DYNAMICS AND VIBRATIONS FOR GEOMETRICALLY NONLINEAR COMPOSITE AIRCRAFT WINGS

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Introduction

Due to their vast advantages, composite materials have been increasingly used during the past several decades in aerospace industry. Recently, the flight vehicles tend to be more flexible and various nonlinearities such as large deflections and nonlinear aerodynamic phenomena, become a major concern for advanced aircraft wing structure. In this regard, it is reported that the modern fighters F-16 and F/A-18 carrying external stores can experience limit cycle oscillations (LCO) [1]. The LCO is unacceptable for flight performance, aircraft store certification and mission capability, and are not predicted in pre-flight linear analysis.

In the present article, the dynamic behavior of a geometrically nonlinear anisotropic thin-walled beam is addressed. Galerkin's method is used for spatial semi-discretization and the multiple scale method [2] will explored for the solution.

Equations of motion

An aircraft wing featuring bi-convex cross-section and with the circumferentially asymmetric stiffness (CAS) lay-up is adopted [3].

$$\delta u_0: a_{14}u_0'' + a_{44}\left(u_0'' + \theta_z'\right) + \underline{\left\{\left[a_{11}v_0' + a_{14}\left(u_0' + \theta_z\right)u_0' + \left(\overset{0}{a_{33}}\theta_x' + \overset{0}{a_{37}}\phi'\right)\phi'\right\}'} - b_1\ddot{u}_0 = 0$$

$$\delta v_0: a_{11}v_0'' + a_{14}\left(u_0'' + \theta_z'\right) + a_{15}\left(w_0'' + \theta_x'\right) - b_1\ddot{v}_0 = 0$$

$$\delta w_0: a_{55}\left(w_0'' + \theta_x'\right) + a_{56}\phi'' + \underline{\left\{\left[a_{11}v_0' + a_{14}\left(u_0' + \theta_z\right)u_0' + \left(\overset{0}{a_{33}}\theta_x' + \overset{0}{a_{37}}\phi'\right)\right]\right\}'} - b_1\ddot{w}_0 = 0$$

$$\delta\phi: a_{37}\theta_x'' + a_{77}\phi'' - a_{56}\left(w_0''' + \theta_x''\right) - a_{66}\phi^{(iv)} + \underline{\left\{\left(\overset{0}{a_{33}}\theta_x' + \overset{0}{a_{37}}\phi'\right)u_0' - \overset{0}{a_{22}}\theta_z' w_0' + \left[a_{81}v_0' + a_{84}\left(u_0' + \theta_z\right) + a_{85}\left(w_0' + \theta_x\right)\right]\phi'\right\}'} - \left(b_4 + b_5\right)\ddot{\phi} + \left(b_{10} + b_{18}\right)\ddot{\phi}'' = 0$$

$$\delta\theta_x: a_{33}\theta_x'' + a_{37}\phi'' - a_{15}v_0' - a_{55}\left(w_0' + \theta_x\right) - \left(b_4 + b_{14}\right)\ddot{\theta}_x = 0$$

$$\delta\theta_z: a_{22}\theta_z'' - a_{14}v_0' - a_{44}\left(u_0' + \theta_z\right) - \left(b_5 + b_{15}\right)\ddot{\theta}_z = 0$$

Solution via the Galerkin method and multiple scale method

Using the Galerkin method, modal expansion[4] and multiple scale method, we get the generalized coordinate form

$$r_i = A_i\left(T_1\right)\exp\left(i\omega_i T_0\right) + c.c$$

where $T_0 = t,\ T_1 = \varepsilon t$ are the time scales, ε is a small parameter, ω_i is the frequency. The internal resonance condition, such as

$$\omega_4 \approx \omega_5 + \omega_6,\ \omega_4 = \omega_5 + \omega_6 + \varepsilon\sigma$$

is introduced, where σ is a detuning parameter. This relationship will lead to the strong interaction between corresponding modes. Then, eliminating the secular terms, we obtain the averaged equations in the complex form

$$2i\omega_4 A_4\left(T_1\right)' = \left(NLT_1\right)A_5\left(T_1\right)A_6\left(T_1\right)\exp\left[-i\sigma T_1\right]$$
$$2i\omega_5 A_5\left(T_1\right)' = \left(NLT_2\right)A_4\left(T_1\right)\bar{A}_6\left(T_1\right)\exp\left[+i\sigma T_1\right]$$
$$2i\omega_6 A_6\left(T_1\right)' = \left(NLT_3\right)A_4\left(T_1\right)\bar{A}_5\left(T_1\right)\exp\left[+i\sigma T_1\right]$$

The amplitude $A_i\left(T_1\right)$ is defined in the polar form

$$A_i\left(T_1\right) = \frac{1}{2}\alpha_i\left(T_1\right)\exp\left[i\beta_i\left(T_1\right)\right] + c.c.$$

Finally, the averaged equations in the polar form are obtained as follows:

$$\omega_4\alpha_1' = -\frac{NLT_1}{4}\alpha_2\alpha_3\sin\left(\gamma\right)$$
$$-\omega_4\alpha_1\beta_1' = \frac{NLT_1}{4}\alpha_2\alpha_3\cos\left(\gamma\right)$$
$$\omega_5\alpha_2' = \frac{NLT_2}{4}\alpha_1\alpha_3\sin\left(\gamma\right)$$
$$-\omega_5\alpha_2\beta_2' = \frac{NLT_2}{4}\alpha_1\alpha_3\cos\left(\gamma\right)$$
$$\omega_5\alpha_2' = \frac{NLT_3}{4}\alpha_1\alpha_2\sin\left(\gamma\right)$$
$$-\omega_6\alpha_3\beta_3' = \frac{NLT_3}{4}\alpha_1\alpha_2\cos\left(\gamma\right)$$
$$where\ \gamma = \beta_1 - \beta_2 - \beta_3 + \sigma T_1$$

where NLT_i is the nonlinear coefficients. These are equations that govern the modulation of their amplitudes and phases of the aircraft wing subjected to given internal resonance condition.

Numerical Result and Discussion

The frequencies and parameters we used here are specified in Table 1.

Fig.1 shows that the responses of α_1, α_2 are almost same, while the response of α_3 is negligible compared to others.

Table 1. Data of the frequencies and parameters.

Parameter	Value
ω_4	4.2351×10^3 rad/s
ω_5	4.1488×10^3 rad/s
ω_6	86.3 rad/s
NLT_1	4.4922×10^3
NLT_2	-4.6691×10^5
NLT_3	0.2101
ε	0.01
σ	0

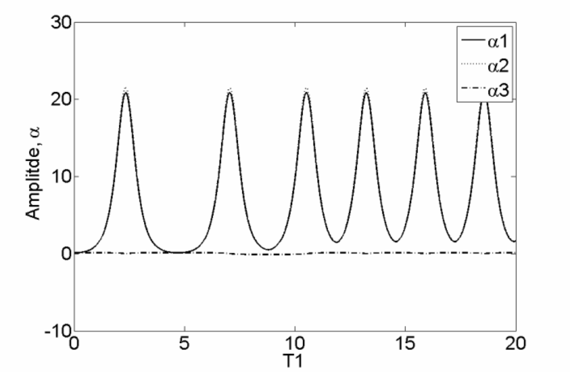

Fig.1 The response of the amplitude α with the initial Condition $\alpha_1(0)=\alpha_2(0)=\alpha_3(0)=0.1$ $\beta_1(0)=\beta_2(0)=\beta_3(0)=0$

In Fig.1, the energy parts in the three directions exchange through the internal resonance. Then, using this result the plunging and pitching displacements are obtained in Fig.2, and Fig. 3.

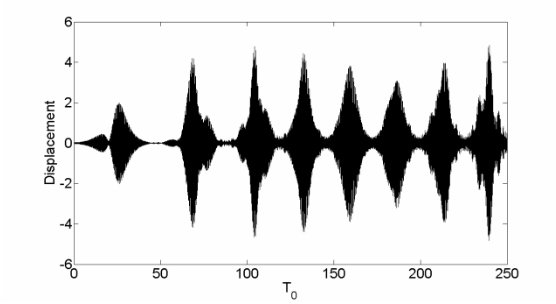

Fig.2 Displacement response of the plunging.

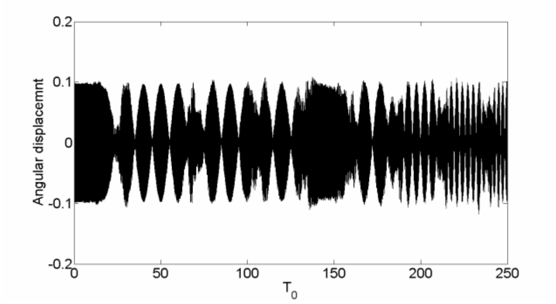

Fig.3 Displacement response of pitching.

Conclusions

We use the multiple scale method to find the internal resonance conditions of the composite thin walled beam, which is caused by coupled nonlinear terms. We also obtain the modulation equations that govern the amplitudes and phase of the interaction modes. Then the nonlinear responses in the case of the internal resonance are investigated. We find the energy components exchange between different directions of motion, which may provide a useful idea for controlling harmful vibrations and making use of vibrations effectively.

References

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