NONLINEAR STATIC ANALYSIS OF A BEAM MADE OF FUNCTIONALLY GRADED MATERIALS

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Introduction

In functionally graded materials (FGM), both microstructure compositions and properties change gradually over the thickness. The idea behind FGMs is to combine two or more different materials into a single composite. For example in aerospace, one could combine aluminum and ceramic together in order to produce a very heat resistant material as well as a material that can withstand very high stresses. Unlike ceramic protection, FGMs use the metal's toughness, thus enhancing the fracture resistance and the life expectancy of the part. Such materials could be used in fields such as heat shields, combustion chambers, nozzles and turbine blades.

Due to their foreseen applications in very-high temperature environments, research on FGMs has covered a wide spectrum of areas such as thermal stress analysis, buckling, stability, vibration analysis. Sankar and Tzeng developed an Euler Bernoulli beam theory for thermal stress analysis without considering nonlinearity[1]. Javaheri and Eslami studied buckling and thermal buckling of FGM plates in which they included higher order terms[2]. These studies used the upper bound of the rule of mixture of composites to estimate the mechanical properties of an FGM. Cho and Ha[3] compared various estimation schemes with finite element method. Prakash and Singha[4] used the Tanaka estimate to determine the locally effective materials properties. Unlike the linear upper bound of the rule of mixture, these estimates take into account the interactions between the matrix and the inclusions.

The gradation patterns in FGMs are typically unsymmetrical, inducing a bending-extension coupling effect similar to that of unsymmetrical composite laminates. Sun and Chin studied large deflection of unsymmetrical crossply laminate under cylindrical bending using von Karman geometric nonlinearity. In this paper, the governing equations for the same problem were applied to FGM beams by taking into account the shear effect.

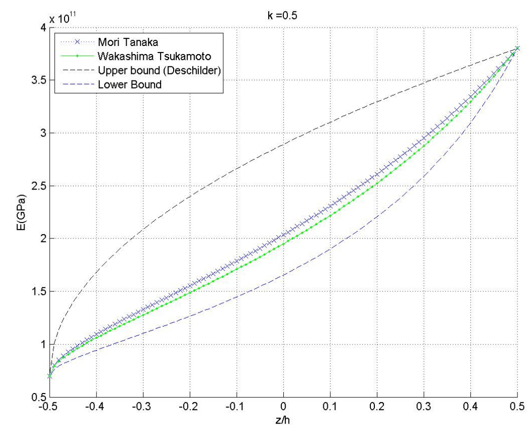
Analytical Model

Young's Modulus

Similar to Eslami and Javaheri[3], the volume fraction of ceramic was defined as the power law:

$$V_c = \left(\frac{2z+h}{2h}\right)^k$$

Classical laminate theory gives a lower and upper bound for the Young modulus, depending on the load directions.



Young Modulus Plotted with Different Schemes

First order shear deformation theory (FSDT)

The displacements for the FSDT are defined as

$$u(x,y,z,t) = u_0(x,y,z,t) - z\phi_x(x,y,t)$$
$$v(x,y,z,t) = v_0(x,y,z,t) - z\phi_y(x,y,t)$$
$$w(x,y,z,t) = w_0(x,y,t)$$

where $\phi_x = \frac{\partial u}{\partial z}$ is the rotation of a transverse normal about the y axis. The governing equation of a FGM beam subject to cylindrical bending can be obtained as

$$\frac{\partial^4 w}{\partial x^4} - \alpha\frac{\partial^2 w}{\partial x^2} = P_1$$

where

$$\alpha^2 = \frac{N_0}{\beta\left(1+\frac{N_0}{k_s G_0}\right)} \quad and \quad P_1 = \frac{P}{\beta\left(1+\frac{N_0}{k_s G_0}\right)}$$

N_0 is the axial load in x-direction, and

$$\beta = \frac{E_2 E_0 - E_1^2}{E_0}$$

The general solution to this 4th order ODE is:

$$w(x) = C_1 + C_2 x + C_3 \cosh(\alpha x) + C_4 \sinh(\alpha x) - \frac{Px^2}{2N_0}$$

where constants C_1 through C_4 depend on the boundary conditions as well as on N_0.

Numerical Results

Simply supported beam

The boundary conditions in this case are $w(\pm a) = 0$ and $M_x(\pm a)=0$, which result in

$$C_1 = \frac{Pa^2}{2N_0} - \frac{E_1 P}{E_0 N_0} - \frac{\beta P}{N_0^2}$$

$$C_2 = 0$$

$$C_3 = \left(\frac{E_1}{E_0} - \frac{\beta P}{N_0^2}\right) \Big/ \cosh(\alpha a)$$

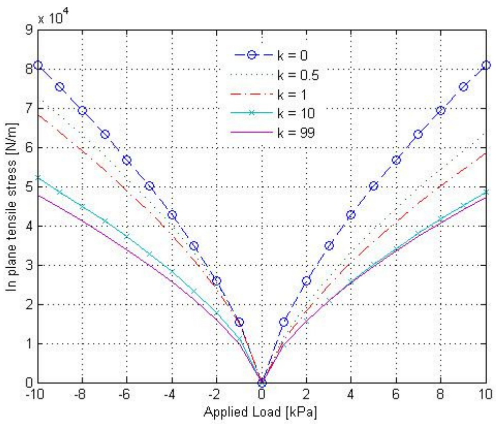
$$C_4 = 0$$

The solution to the governing equation becomes

$$w(x) = C_1 + C_3 \cosh(\alpha x) - \frac{Px^2}{2N_0}$$

and N_0 can be obtained as

$$N_0 = \frac{E_0}{4a} I_1 + \frac{E_1 (k_s G_0 + N_0)}{2ak_s G_0} I_2 - \frac{2Pa}{k_s G_0}$$



In-plane Tensile Stress vs. Applied Load

Clamped beam

For a beam clamped at both ends, $w(\pm a) = 0$ and $w_{,x}(\pm a) = 0$. The general solution to the governing equation becomes

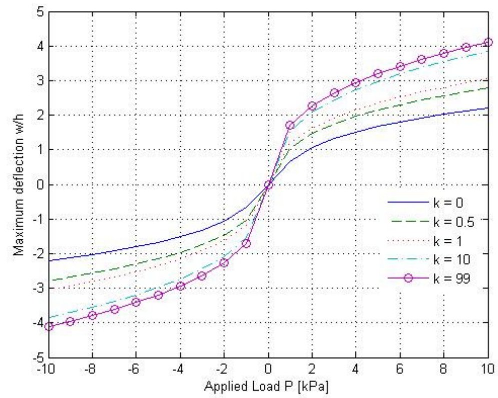
$$w(x) = C_1 + C_3 \cosh(\alpha x) - \frac{P}{2N_0} x^2$$

where

$$C_1 = \frac{Pa^2}{2N_0} - \frac{Pa}{\alpha N_0} \frac{\cosh(\alpha a)}{\sinh(\alpha a)}$$

$$C_2 = C_4 = 0$$

$$C_3 = \frac{Pa}{\alpha N_0} \frac{1}{\sinh(\alpha a)}$$



Maximum deflection of the beam under cylindrical bending with various k values

Conclusions

The following conclusions were reached based on the current studies:

(i) There exists coupling between the bending moment and the in plane tensile stress.

(ii) When k tends to either or infinity the beam's behavior will become akin to that of an isotropic beam made of the associated component.

(iii) Due to their asymmetric nature, the nonlinear effects cannot be neglected when studying FGM beams.

References

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