A SEMIDEFINITE HARMONIC OSCILLATOR FOR SEISMIC ANALYSIS OF STRUCTURES IN A COMPLEX SPACE. A PHYSICAL AND MATHEMATICAL SHAKE TABLE.

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Introduction

In the design of engineering structures the mathematical models (not physical) assume boundary conditions that guarantee the stability of any structure. The nature of the frontier is implicit in the condition that we impose on the model (in real variable) as being positive definite. So any dynamic analysis focuses all the attention on the interior of the structure disregarding the vibrations transmitted to or from the boundary. The main assumption is that the base displacement is zero[1].
Many attempts have been made at solving the soil-structure interaction problem; however, in the end the mathematical model of a structure under the action of an earthquake is replaced by a set of equivalent forces proportional to the product of the masses m_i and the ground acceleration $\ddot{u}_g$. For three-dimensional dynamic equilibrium we have[2]:

$$\mathbf{M}u + \mathbf{C}u + \mathbf{K}u = \mathbf{m}_x v_x + \mathbf{m}_y v_y + \mathbf{m}_z v_z \qquad \textbf{(A)}$$

Where, **M**, **C** and **K** are the mass, damping and stiffness matrices of the structure model. The relative displacements **u**, that exist for the soil are set equal to zero at the sides and bottom of the foundation. The terms v_x, v_y and v_z functions of the time variable t are the free-field components of the acceleration, if the structure is not present. The columns of matrices $\mathbf{m_i}$ are the directional masses for the added structure.
The results have not been satisfactory and so (R. Englekirk see ref[1]) for a currently performed test of a building model on a shake table: ***"The direct correlation between a model that is one tenth the size of a building and one that is full size is speculative at best"***. This revealing statement was made to accept the limitations of present knowledge and to justify the construction of an enormous shake table 25 feet by 40 feet, recently built in the USA. In this dilemma, we will see that the only way to solve this problem is to resort to the semidefinite model of dynamic analysis recently[1] developed in a complex space and to include the use of the concept of ***"Seismic Analysis"*** via a semidefinite model that includes a ***mobile finite o infinite half-space*** that supports ***one or multiple degrees of freedom***.

Mathematical and Physical Model

Figure 1 shows a semidefinite system with four masses $\boldsymbol{m_0}$, $\boldsymbol{m_1}$, $\boldsymbol{m_2}$ and $\boldsymbol{m_n}$ coupled with springs k_1, k_2 to k_n and with dampers c_1, c_2 to c_n. Mass m_0 stands for the support of the main system m_1, m_2, to m_n.

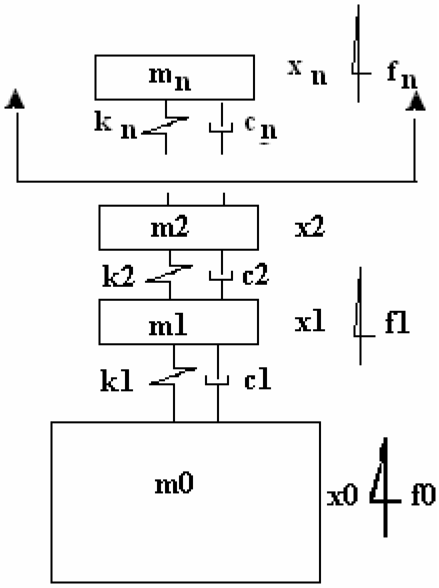

Figure 1

The model of Fig. 1 becomes a classical positive definite system with **n** degrees of freedom when the mass m_0 is either much larger than the other three masses or when it becomes infinite. Rigid body motion is possible[1].

Motion Equation

$\boldsymbol{Z_{mn}X_n = F_n}$ (1)

For a four masses case equation (1) looks as follows:

$$\begin{bmatrix} -\omega^2 \cdot m_0 + i\cdot\omega\cdot c_1 + k_1 & -i\cdot\omega\cdot c_1 - k_1 & 0 & 0 \\ -i\cdot\omega\cdot c_1 - k_1 & -\omega^2\cdot m_1 + i\cdot\omega\cdot(c_1+c_2)+k_1+k_2 & -i\cdot\omega\cdot c_2 - k_2 & 0 \\ 0 & -i\cdot\omega\cdot c_2 - k_2 & -\omega^2\cdot m_2 + i\cdot\omega\cdot(c_2+c_3)+k_2+k_3 & -i\cdot\omega\cdot c_3 - k_3 \\ 0 & 0 & -i\cdot\omega\cdot c_3 - k_3 & -\omega^2\cdot m_3 + i\cdot\omega\cdot c_3 + k_3 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

Where X_n and F_n are in general functions of the complex variable $(a+bi)$ and $i = \sqrt{-1}$.

Results and Discussion

A three masses structural system, according to nomenclature of Fig. 1, assumes the following parameters: masses m_0 = 1000000, m_1 = 4.97 and m_2 = 4.82 MU (Mass Units), springs k_1 = 258.5, k_2 = 150.2 Tons/m and damping of $\xi_1 = \xi_2 = 0.05$ i.e. 5% of the critical one. Forcing force F_0 = 1000000 Tons such that we apply an effective acceleration $\mathbf{a} = (F_0 / m_0) = 1$. The September 19, 1985 E-W component accelerogram recorded at the SCT site in Mexico city shown in Fig. 2.

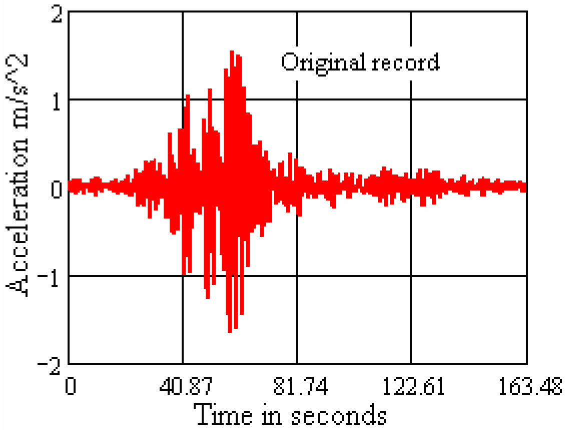

Figure 2, Sept 19 1985, SCT E-W accelerogram.

Equation (1) was programmed in the computer code MathCad 13. The solutions X_j for j=0 to2 are complex quantities with the magnitudes $|X_j|$ and the phase angles θ_j encoded in them. The resulting transfer functions $|X_j|$ of the three masses under unit acceleration at the base m_0 are shown in Figure 2 along with the frequency spectrum of the SCT record used in the dynamic analysis.

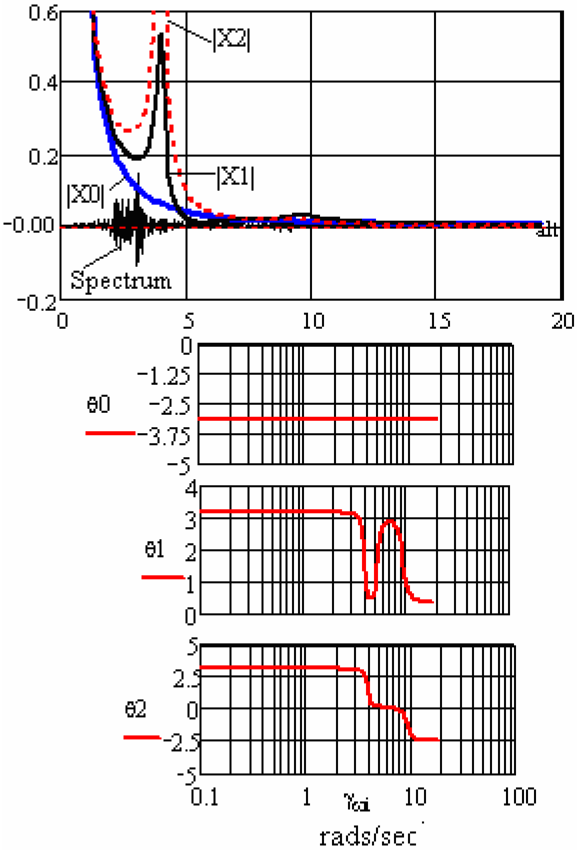

Figure 3, Spectrum, transfer functions | $X_{jm}(\gamma_{\omega i})$| and Phase angles.

It is noticeable that the X_0 displacement never changes phase θ_0 for any frequency $\gamma_{\omega i}$. However, as the excitation frequency $\gamma_{\omega i}$ changes the other two displacements change the phase angles θ1 and θ2 progressively in order to satisfy dynamic balance[1].

The ordinates B_j of the spectrum of Figure 2 are multiplied by the ordinates the three displacements $|X_j|$ and the final displacements were obtained via the following equation in a Fourier analysis scheme:

$$X_j(t) = \sum_{m=1}^{800} \left[\left| X_{jm}(\gamma_{\omega i}) \right| \cdot B_m \cdot \sin\left(\frac{\pi}{L} \cdot m \cdot t + \theta_m \right) \right] \quad (2)$$

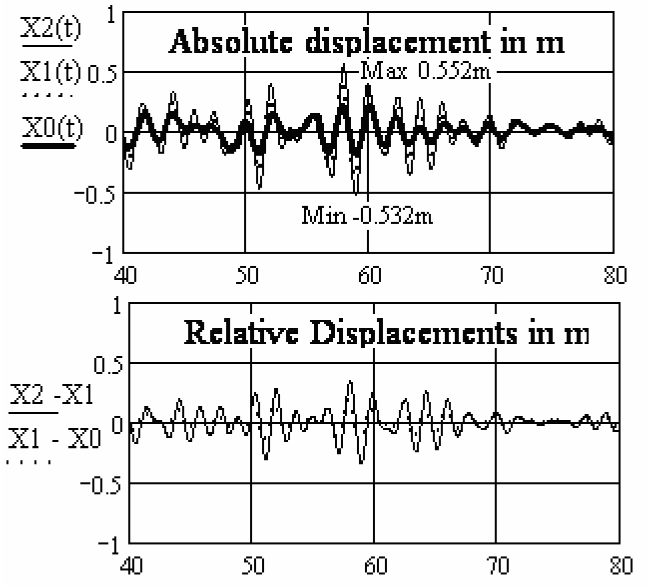

Figure 4, Absolute and relat. displ. in meters.

Differentiating equation twice with respect to the time variable t and plotting from o to 160 seconds leads to Figure 5 as follows:

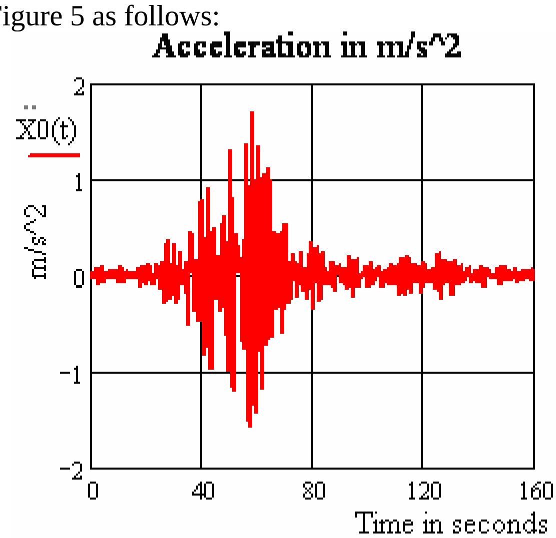

Figure 5, Eqn(2) differentiated twice with respect to time variable t.

That recovers an approximation of the acceleration input of Figure 2 and proves that results of Fig.4 are correct. The small differences are due to the interaction with the masses m_1 *and* m_2 and this should be reproduced with a high degree of accuracy on a shake table.

References 1.) J.L. Urrutia-Galicia, Contr. to Appl. Math. in Engineering II. Paper # 2, The semidefinite model for hysteretic nonlinear elastic vibrations of structures. "Dynamic balance, the second Newtonian Law and the Reciprocity Theorem", by J. L. Urrutia-Galicia. **Berichte aus dem Konstuktiven Ingenieurbau, Universitat der Bundeswehr München, Deutschland (Germany), No. 08/1, 2008.** 2.) E. L. Wilson, Three-Dimensional Static and Dynamic Analysis of Structures. A physical Approach. With emphasis on Earthquake Engineering. Computers and Structures Inc., Berkeley, California, USA, 3rd Editon, January 2002, pag. 16-5, eqns. (16.5) and (16.6).