MAGNETO-ELASTIC RESPONSE OF RECTANGULAR ORTHOTROPIC PLATES DUE TO MULTI-LANE MOIVNG LOADS

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Introduction

The progress of electro-magnetic technology opens up a venue of applications of magneto-elastic mechanics to maglev system. The advance of material science promotes interdisciplinary field applying the properties of matter to various areas of engineering. Orthotropic plate structures have played important applications to mechanical engineering for their different material properties in two mutually perpendicular directions. In this paper, attention will be focused on dynamic analysis of an orthotropic rectangular plate subject to multi-lane moving loads in uniform magnetic filed. Of particular interest is to derive an analytical solution for the dynamic response of the orthotropic plate subject to the simultaneous action of a moving load and uniform magnetic field.

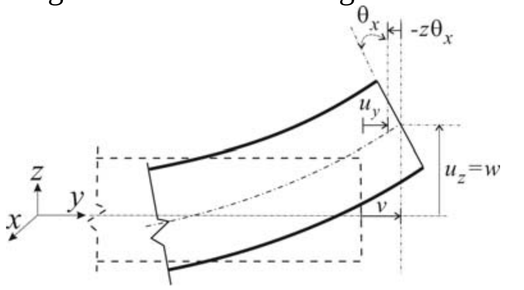

Fig. 1 Displacements of a generic point in the *y-z* plane;

Formulation of the Problem

By considering the assumption of Kirchhoff's thin plate theory, the displacements (u_x ,u_y, u_z) of a generic point at section (x, y) in a plate can be expressed in terms of the mid-surface displacement components (u, v, w) along the coordinate axes (x, y, z) of the plate as are expressed as

$$\begin{aligned} u_x(x,y,z) &= u + z\theta_y = u(x,y) - z(\partial w/\partial x), \\ u_y(x,y,z) &= v - z\theta_x = v(x,y) - z(\partial w/\partial y), \\ u_z(x,y,z) &= w(x,y). \end{aligned} \tag{1}$$

With the displacement fields of Eqs. (1), the constitutive equations of stress-strain relationship for an orthotropic rectangular plate are expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} E_{xx} & E_{xy} & 0 \\ E_{xy} & E_{yy} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}, \tag{2}$$

where E_{xx} and E_{yy} are the elastic moduli along x and y directions, respectively, and G_{xy} the shear modulus. They are given as follows [1]:

$$\begin{aligned} E_{xx} &= \frac{E_1}{1-\upsilon_{12}\upsilon_{21}}, E_{yy} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}}, \\ E_{xy} &= \frac{\upsilon_{12}E_1}{1-\upsilon_{12}\upsilon_{21}}, G_{xy} = G_{12}, \upsilon_{12}E_2 = \upsilon_{21}E_1. \end{aligned} \tag{3}$$

Here, the subscript "1" means the x-coordinate, "2" the y-coordinate, and $(\upsilon_{12},\upsilon_{21})$ = Poission ratios. Hence, E_1 and E_2 mean elastic moduli in 1– and 2– directions, respectively, and G_{12} = shear modulus.

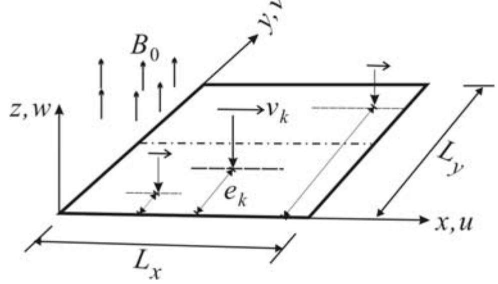

Fig. 2 A rectangular orthotropic plate under multi-lane moving loads.

Governing equations

As shown in Fig. 2, a rectangular orthotropic plate is subjected to multi-lane moving loads in magnetic field. The governing equation of the rectangular orthotropic plate under multi-lane moving loads in uniform magnetic field is [2,3]:

$$\rho h\ddot{w} + D_{11}w_{,xxxx} + 2Hw_{,xxyy} + D_{22}w_{,yyyy} - \Gamma(\dot{w}_{,xx} + \dot{w}_{,yy}) = \sum_{k=1} p_0\delta(x - v_k t)\delta(y - e_k) \quad (4)$$

where ρ = mass density of plate, h = thickness, e_k = distance between the kth path along the x-direction, and $H = D_{12} + 2D_{66}$,

$$D_{11} = \frac{E_1 h^3}{12(1-\upsilon_{12}\upsilon_{21})}, D_{12} = \upsilon_{21}D_{11}, \quad D_{22} = \frac{E_2}{E_1}D_{11}, D_{66} = \frac{G_{12}h^3}{12}, \Gamma = \frac{\mu h^3}{12}B_0^2 \quad (5)$$

and μ = electric conductance and B_0 = magnetic flux density.

Solution

For a simply supported rectangular plate, the transverse deflection w can be expressed as

$$w(x,y,t) = q_{mn}(t)\sin\left(\frac{m\pi x}{L_x}\right)\sin\left(\frac{n\pi y}{L_y}\right) \quad (6)$$

By Galerkin's method, the equation of motion in Eq. (4) can be converted into the following generalized equation

$$\rho h\ddot{q}_{mn} + B_{mn}\dot{q}_{mn} + \rho h\omega_{mn}^2 q_{mn} = \frac{4p_0}{L_x L_y}\sum_{k=1}\sin\frac{m\pi v_k t}{L_x}\sin\frac{n\pi e_k}{L_y} \quad (7)$$

where

$$B_{mn} = \frac{\Gamma\pi^2}{L_x^2}\left(m^2 + (n/\lambda)^2\right), \quad \omega_{mn}^2 = \frac{\pi^4}{\rho h L_x^4}\left[m^4 D_{11} + (n/\lambda)^4 D_{22} + 2(mn/\lambda)^2 H\right] \quad (8)$$

where $\lambda = L_y / L_x$. With zero initial conditions, i.e., $w(x,y,0) = \dot{w}(x,y,0) = 0$, as the multi-lane moving loads run toward the x-direction, the displacement of the plate under the forced vibration of the moving load can be obtained by using Duhamel's rule:

$$w(x,y,t) = \sum_{m=1}\sum_{n=1} q_{mn}(t)\sin\frac{m\pi x}{L_x}\sin\frac{n\pi y}{L_y}$$

$$q_{mn}(t) = \frac{p_0}{\varpi_{mn}\rho h}\sum_{k=1}\left[\sin\frac{n\pi e_k}{L_y}\times \int_0^t e^{-\xi_{mn}\omega_{mn}(t-\tau)}\sin\frac{m\pi v_k\tau}{L_x}\sin\left(\varpi_{mn}(t-\tau)\right)d\tau\right] \quad (9)$$

where

$$\xi_{mn} = \frac{B_{mn}}{2\rho h\omega_{mn}}, \varpi_{mn} = \omega_{mn}\sqrt{1-\xi_{mn}^2} \quad (10)$$

With the analytical result shown in Eqs. (9) and (10), parametric studies will be performed for a rectangular orthotropic plate under the simultaneous action of moving loads and uniform magnetic force. Fig. 3 shows the time-history deflection responses of a square orthotropic plate subject to two-lane moving loads with different speeds in uniform magnetic field.

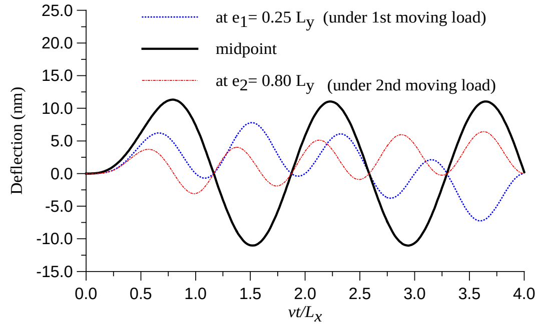

Fig. 3 Time history deflection response of the orthotropic plate under two-lane moving loads.

Conclusion

This study adopted the orthotropic plate theory and electro-magnetic mechanics to investigate dynamic response of a rectangular orthotropic plate subject to multi-lane moving forces analytically. The analytical solution can be used as a benchmark solution in calibrating the predicted results by numerical methods.

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