# THEORETICAL ANALYSIS AND NUMERICAL MODELING OF ELASTIC WAVE PROPAGATION IN HONEYCOMB-TYPE PERIODIC STRUCTURES

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## Introduction

Honeycomb-type periodic structures are widely used for their good physical and mechanical properties. Therefore interests of understanding the dynamic wave behaviors of them have increased. Generally homogenized models are used by neglecting the micro-structural details and using an equivalent constitutive law with continuous mechanical characteristics instead [1, 2]. The classical homogenized models usually offer an efficient and reliable solution to investigate the static or low frequency (LF) dynamic behaviors of the periodic structures. However our previous works show that for the high frequency (HF) flexural wave propagation in the honeycomb-type thin layers, the homogenized models fail to give appropriate simulation results [3, 4].
Indeed, for the HF ranges, the involved wavelengths are as short as or even shorter than the cell's characteristic lengths, so interactions between the waves and the periodic cells become important and result in complex deformations of cellular walls, which cannot be taken into account by the classical homogenized models. Hence more appropriate modeling should consider and integrate the effects of the microstructure of the periodic structures. However, a model that describes every detail of the microstructure of periodic structures is very expensive or even prohibitive for large structures. Therefore, some recent studies applied the Bloch wave theorem to investigate the wave propagation in periodic structures, benefit from which the domain to be modeled can be reduced from the entire periodic structure to a primitive cell [5].
According to the Bloch wave theorem, any non-periodic function, $V(\mathbf{x})$, defined on a periodic structure having $Q_0$ as the primitive cell, can be decomposed into its Bloch wave modes $V^B(\mathbf{x}, \mathbf{k})$, with $\mathbf{k}$ the wave vector restricted in the reciprocal cell $Q_0^*$ [6]. $V^B(\mathbf{x}, \mathbf{k})$ are periodic functions and have the same periodicity as the periodic structure. In other word, the wave propagation phenomena through the whole structure can be understood by investigating the Bloch wave modes in the sole primitive cell, so lots of efforts can be saved when doing analysis and simulation.
The present work is devoted to the theoretical analysis and numerical modeling of elastic wave propagation in one-dimensional (1D) rod and two-dimensional (2D) beam rectangular and hexagonal periodic structures by using the Bloch wave theorem.

## Elastic wave propagation in a 1D periodic elastic rod structure

At first, a 1D periodic structure made of elastic rods is considered. Its primitive cell is composed of two rigid-jointed elastic rods.
By solving the Bloch equilibrium eigenequation in the primitive cell, we get, $\forall k \in Q_0^*$, the Bloch eigenmodes, $U^B(x, k)$, corresponding to the eigenfrequency, $\omega$. For each rod, $U^B(x, k)$ contains two constants that need to be determined. Then, by writing the interface conditions between the two rods at their junction point and periodic conditions at the ends of the primitive cell, a system with four linear equations to solve the constants is obtained. To ensure that the system admits nontrivial solutions, its determinant should vanish, which finally gives rise to the dispersion relation between k and $\omega$. In the 1D case, the dispersion equation can be got analytically and then the frequency band-gap is plotted numerically (Fig. 1).

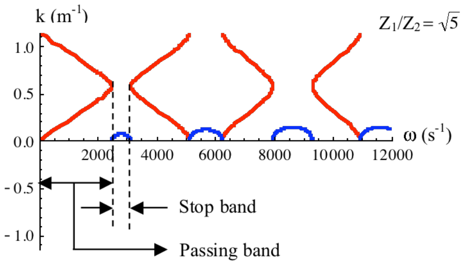
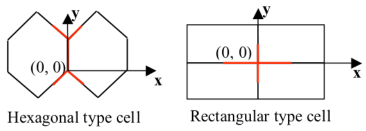


Fig. 1 Frequency band-gap of 1D rod structure

We remark that the location and the width of the stop bands strongly depend on the ratio between the characteristic acoustic impedances $Z_1$ and $Z_2$ of the two rods.
With the Bloch eigenmodes that we calculated, the diffracted waves caused by the periodic cells can be evaluated. We observe an important amplification phenomenon of wave amplitude due to the diffraction by the cellular microstructure of the periodic structure.

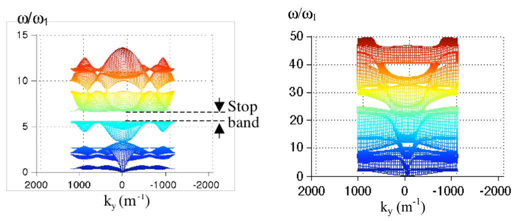
**Elastic wave propagation in 2D periodic elastic beam structures**

2D elastic beam periodic structures with respectively hexagonal and rectangular cells are also analyzed. Here, the thickness of the vertical beams is the twice the one of the other beams (Fig. 2).



(a) (b)
Fig. 2 2D elastic beam periodic structures

Like in the 1D case, we apply the Bloch wave theorem to obtain the frequency band-gaps of the structures. We find out that the first frequency stop band of the hexagonal structure appears at the frequency that is about five times the first pinned-pinned flexural resonance frequency of the beam, $\omega_1$. For the rectangular structure, no frequency band-gap exists in the whole frequency domain, whatever a/b the ratio between the beam lengths (Fig. 3).



(a) Hexagonal (b) Rectangular, a/b=2
Fig. 3 Dispersion surfaces

## Conclusion

In this work, the Bloch wave theorem is applied to the study on elastic wave propagation in the 1D and 2D periodic structures. The dispersion relation is obtained analytically and numerically for the 1D rod structure. The diffracted waves triggered by the periodically placed cells and the amplification of wave amplitudes are observed. For the 2D periodic structures with hexagonal or rectangular cells composed by elastic beams, the dispersion relation is analyzed numerically and the frequency band-gaps are obtained.

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